Debunking the Myth of Southern Profligacy. A DSGE Analysis of Business Cycles in the EMU's Big Four*

Alice Albonico†
University of Milano-Bicocca

Roberta Cardani‡
University of Pavia

Patrizio Tirelli§
University of Milano-Bicocca and Griffith University

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Abstract

We uncover important asymmetries in the drivers of EMU big four's business cycles. In Germany and France demand shocks played a major role in determining output growth before and after the financial crisis. In Spain demand shocks were far less important and we find evidence of favourable technology shocks before the financil crisis. The sovereign bond crisis was followed by a sequence of adverse permanent technology shocks both in Spain and in Italy. These latter results are consistent with recent theoretical developments that emphasize the adverse supply-side effects of a credit crunch.

Keywords: *Asymmetric Euro crisis, two-country DSGE, Bayesian estimation*
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†University of Milano-Bicocca, Piazza dell'Ateneo Nuovo 1, 20126 Milano, Italy. Email: alice.albonico@unimib.it.

‡University of Pavia, via San Felice, 7 27100 Pavia, Italy. Email: roberta.cardani@unipv.it.

§University of Milano-Bicocca, Piazza dell'Ateneo Nuovo 1, 20126 Milano, Italy. Email: patrizio.tirelli@unimib.it.

1 Introduction

The creation of the European Monetary Union (EMU) in 1999 was heralded as the beginning of a new phase of prosperity and relatively fast growth, potentially leading to convergence between peripheral and core countries. The early EMU years seemed to confirm this prediction, but the sovereign bond crisis severely hit the "peripheral" members and raised concerns of a possible breakup of the Euro Area, due to the persistent growth gap between the core and the periphery. These patterns are well exemplified by output growth dynamics in the four largest EMU countries - France, Germany, Italy and Spain (Figure 1), which account for 79% of the Euro Area GDP. After the relatively favorable $1999-2006$ period, growth rates in Italy and Spain plunged and remained negative until the end of the sample. While the $2008-2009$ financial crisis was a generalized phenomenon which affected all developed economies, the sovereign bond crisis has been specific to the Eurozone, apparently rooted in the imbalances that accumulated in the pre-2008 period.

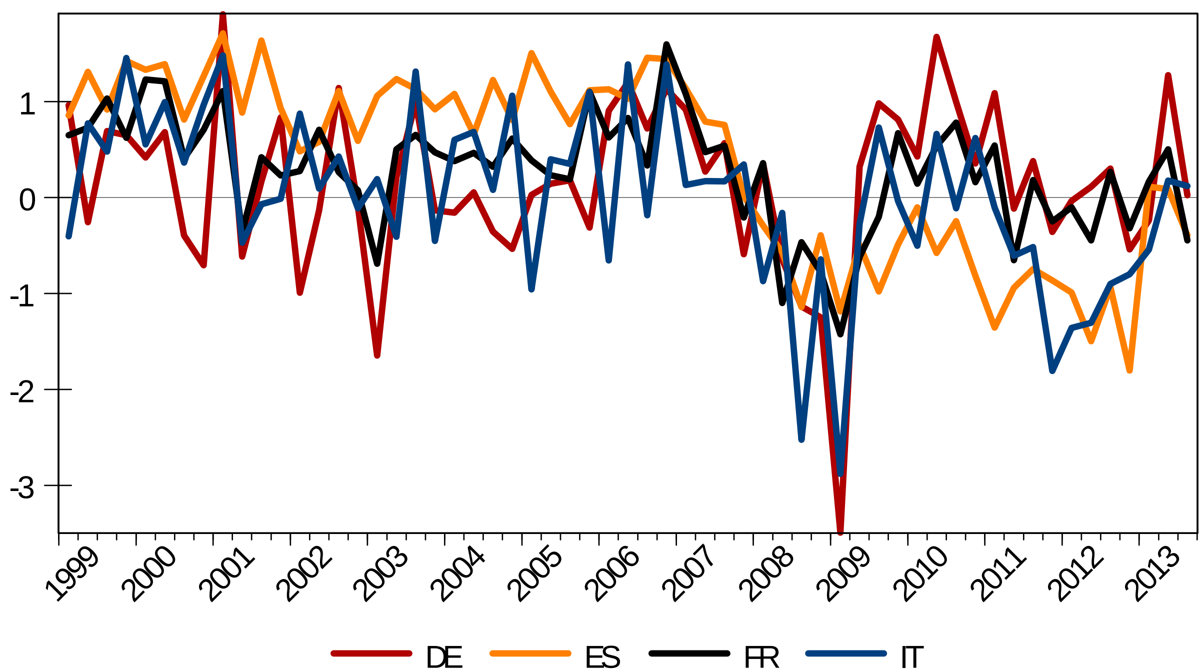

Figure 1: GDP growth - EMU largest economies

In peripheral countries the early EMU years had been characterized by relatively loose domestic credit conditions, and by real exchange rate appreciations. These phenomena were initially seen as part of the catching up process triggered by monetary integration (Blanchard and Giavazzi, 2002). After the onset of the 2010 sovereign bond crisis a "new view" has suggested that favorable credit conditions triggered a surge in consumers demand and, by falling short of stimulating productivity convergence, determined demand-driven erosion of competitiveness in these countries (Giavazzi and Spaventa, 2011; Sinn, 2011; Fernandez-Villaverde et al., 2013). The new view also implied that to get out of the crisis policymakers should seek internal devaluations, mainly through fiscal austerity programs and labor market reforms.

It should be noted, however, that the 2010 sovereign crisis was triggered by (potentially self-fulfilling) fear that a Grexit domino effect would cause EMU disintegration (De Grauwe and Ji, 2013). In the periphery this caused a severe credit crunch that might have produced the growth slowdown, in analogy with what is typically observed in consequence of "sudden stops" (Furceri and Mourougane, 2012; Benigno et al., 2015). In figure 2 we report descriptive statistics of total, non-financial business and mortgage credit cycles for France, Germany, Italy and Spain, as computed in Samarina et al. (2017). For Spain and Italy they document a large credit financial boom between 2005 and 2008, more pronounced for the business than for the mortgage sector. The subsequent post-2010 contraction mainly hit the business sector. By contrast, the credit contraction was relatively limited in France, and there is no evidence of a credit boom/bust cycle in Germany.

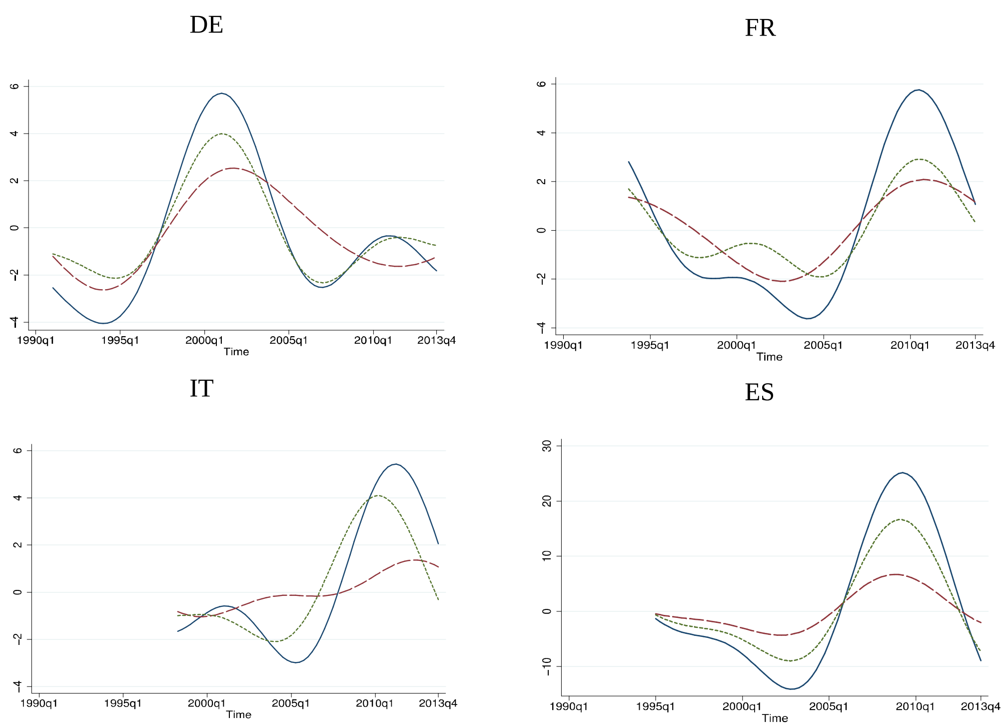

Figure 2: Total credit cycles (solid lines), mortgage credit cycles (dashed lines), and non-financial business credit cycles (dotted lines). *Source: Samarina et. al (2016)'s calculations.*

Theoretical contributions have explored the supply-side effects of a credit crunch. Khan and Thomas (2013) show that financial shocks penalize firms characterized by relatively high productivity but relatively low net worth, thereby causing reductions in aggregate total factor productivity. Bassetto et al. (2014) argue that a credit crunch has a particularly severe effect on small firms production. Buera et al. (2015) obtain a similar result focusing on employment dynamics. This alternative view emphasizes that the supply-side effects of a credit crunch bear important policy implications. In fact, Laeven and Valencia (2013) find that firms exposed to external finance greatly beneficed from bank recapitalization plans and fiscal policies designed to stimulate domestic demand.

The paper investigates the drivers behind business cycle dynamics in EMU four largest economies. One may expect this model to answer two questions which are crucial to the definition of policies designed to restore growth in the southern economies. Was the pre-2007 relatively fast growth in the south driven by demand shocks or was there also some favorable productivity component? How can we explain the post-2010 dismal performance of these countries? Was it due to lack of domestic demand, including contractionary fiscal policies, or was it caused by a slowdown in the underlying rate of productivity growth?

We consider a number of country-specific technology and demand shocks. Technology shocks include standard temporary TFP shocks and shocks to the productivity growth trend, entailing for each country permanent variations in productivity levels relative to the rest of the Eurozone. Non-policy demand shocks include "risk premium" shocks driving a wedge between the return on capital accruing to the households and the price of capital services paid by firms, and standard investment-specific shocks.

Our results challenge the view that loose domestic (private and/or public) demand conditions in the early EMU years are at the root of the Eurozone crisis. We find no evidence of a large demand-driven boom in Italy or Spain before the financial crisis, while in this period favorable demand shocks played an important role in Germany and France. It is interesting to note that the hypothesis of a credit- (demand) driven boom is also rejected by Chouard et al. (2014), who use a reduced-form equation of total factor productivity dynamics. As for fiscal policy, we cannot find evidence of an expansionary bias in discretionary public consumption in southern economies. In fact the contribution of fiscal shocks in Spain is comparable to what we observe for France or Germany, whereas in Italy the fiscal shocks were larger but did not induce a systematic expansion.

Since 2008 demand shocks remain crucial to explain the initial slump and the subsequent post-2010 modest recovery in Germany and, to a lesser extent, in France. Adverse permanent technology shocks explain the post-2008 severe output losses observed for Italy. The same conclusion applies to Spain after 2010. Thus, the severe output contractions in Italy and Spain should not be interpreted as the necessary correction of accumulated imbalances. They signal instead a North-South divide, determined by a permanent technology gap. This is broadly consistent with the view that the crisis is the consequence of the supply side effects of the banking crisis in the South. Ball (2014) estimates the effects of the financial crisis on potential output in a panel of OECD countries, including the four countries considered here. It is interesting to note that his ranking of the financial crisis effects fits very well our results. Potential output in Germany was virtually unscathed, France suffered a break in potential output growth of relatively limited importance, a severe disruption is observed for Italy and Spain.

The paper is organized as follows. Section 2 motivate our focus on permanent technology shocks highlighting the connections between our theoretical and empirical modelling strategies. Section 3 introduces the estimation strategy and section 4 presents the results. Finally, section 5 concludes.

2 Modelling strategy

Right from the outset, it is worth to emphasize the connections between our theoretical and empirical modelling strategies. The empirical DSGE literature incorporating non-trivial financial frictions and a banking sector has been rapidly expanding since the outset of the financial crisis. Gerali et al. (2010) find that financial shocks contributed to explain the output fall during the 2007 financial crisis, but in their model a bank capital loss cannot replicate the amplitude of the $2007-2008$ downturn. Brzoza-Brzezina and Kolasa (2013) evaluate the performance of DSGE models incorporating the two main approaches to modelling financial frictions. According to the first one, frictions are due to costly monitoring and materialize in the form of credit spread (Bernanke, Gertler, and Gilchrist, 1999; Christensen and Dib, 2008; Christiano, Motto and Rostagno, 2010). According to the second one, agents heterogeneity introduces a distinction between lenders and borrowers, and borrowers are required to provide collateral for their loans (Kiyotaki and Moore, 1997; Gerali et al. 2010). They find that modelling financial frictions is essential for replicating fluctuations in financial variables, but this is not sufficient to improve over the statistical fit of the workhorse New Keynesian model, such as Smets and Wouters (2007). An identical conclusion is reached in Suh and Walker (2016) and Lindé, Smets and Wouters (2016).

It is also important to notice that recent empirical DSGE models of the financial crisis utterly neglect the potential role of permanent productivity shocks. Gerali et al. (2010) apply an HP filter to trending variables; others, such as Suh and Walker (2016) and Lindé, Smets and Wouters (2016) impose a deterministic trend on these variables. One standard justification for this approach is that the low frequency features of data series bear relatively negligible importance for empirical models that focus on the short-term behavior of the economy.

In sharp contrast with these studies, Sims (2011) emphasizes the importance of jointly considering the roles of the persistent but transitory productivity shocks of the RBC-DSGE literature and of the permanent shocks identified in the VAR literature (Galí, 1999). He shows that incorporating permanent technology shocks in an empirical DSGE model bears important implications for estimated parameters. In a nutshell, he claims that incorporating permanent shocks allows to achieve a better match of the empirical responses to technology shocks with fewer frictions than in standard empirical DSGE models.

Finally, one common feature of the financial frictions modelled in the above mentioned contributions is that a credit crunch affects demand for consumption and investment goods, but it has no effect on the growth rate of productivity.

Given that our objective is to identify the distinct role played by demand factors and variations in productivity growth during the two crises, and in the light of the apparent difficulty of improving the empirical performance of DSGE models by explicitly adding financial frictions, we have chosen to follow Kollmann et al. (2016) who do not explicitly model financial frictions but incorporate permanent technology shocks in an otherwise standard DSGE model to analyze the post-2007 different macroeconomic performances of the Euro Area and of the US. Consistently with our results, they find that the relatively bad performance of the Euro Area reflects a combination of adverse aggregate demand and supply shocks, including permanent TFP shocks.

In concluding this discussion it is important to stress the different approaches to the measurement of permanent TFP shocks relative to more conventional studies. The early RBC literature focused on persistent but transitory changes in the measured Solow residual as a source of technology shocks. This approach is criticized because the procyclical behavior of the Solow residual may be due to cyclical errors in measuring changes in capital utilization and/or in the intensity of work effort (Basu, 1996). As a consequence, Basu et al. (2006) advocate the adjustment of the Solow residual, controlling for imperfect competition and time varying utilization of capital and labor. Sims (2011) shows that the Basu et al. TFP measure incorporates both temporary but persistent and permanent shocks. Our approach is obviously different because we estimate temporary and permanent technology shocks jointly with a number of temporary demand and markup shocks, as it is typical of the empirical DSGE literature. Instead of being treated as a residual, in our framework technology shocks are identified on the grounds of their ability to explain permanent comovements of observed variables, including permanent variations of the capital labor ratio and of consumption levels. Our contributions is therefore quite distinct from earlier work on TFP in the Eurozone (Cette et al., 2016; Gamberoni et al. 2016).

Our empirical strategy defines the theoretical characterization of a monetary union economy.

As we discuss in section 3 below, we estimate a two-country monetary union model for each of the four countries. This option is preferred to the alternative of estimating a multi-country model for the whole Eurozone, which would be intractable.[1] We therefore assume that the monetary union is composed of the domestic economy (D, size s) and of the rest of the Euro Area economy (REA, size $1-s$). In what follows we describe the D economy, as the REA economy is characterized symmetrically. An asterisk identifies variables and parameters referring to the REA economy.

Each region produces both non-tradable and tradable goods. Following Rabanal (2009), there is no price discrimination across regions, i.e. the law of one price holds. The structure of the economy in each region is very close to Christoffel et al. (2008). Households supply factor inputs to monopolistic producers of intermediate goods and delegate wage setting decisions to monopolistic labor unions. Retail firms demand intermediate goods to assemble the final consumption and investment bundles under perfect competition. The model features standard nominal and real frictions, i.e. price and nominal wage stickiness modelled *à la* Calvo, investment adjustment costs, variable capacity utilization, external consumption habits.

There is a continuum of households indexed by i. As in Kollmann et al. (2016) we draw a distinction between a fraction $1-\theta$ of Ricardian households ($i=o$) and the remaining θ Non-Ricardian or rule-of-thumb households ($i=rt$). Non-Ricardian households do not have access to financial markets and consume all their disposable labor income in each period. Ricardian households participate in financial markets, trade government bonds, accumulate physical capital and own firms.[2]

The model incorporates an exogenous fiscal sector, including public consumption, tax rates on factor incomes, transfers and lump sum taxes. Factor incomes taxation and transfers are assumed constant and used to calibrate relative consumption between the two households groups as in Coenen et al. (2013). Public consumption is one among the observables used to estimate the model and we assume its cyclical pattern is driven by exogenous shocks whereas lump-sum taxation of Ricardian households ensures government solvency.

The technical Appendix provides a full description of the model. In what follows we focus on certain aspects of the model that are crucial to understand our results, i.e. characterization of preferences and shocks.

The representative firm producing intermediate goods uses the following production technology:

$$Y_t^{int} = \varepsilon_t^{a,int}[u_t^{int}K_t^{int}]^{\alpha_{int}}[z_t h_t^{int}]^{1-\alpha_{int}} - z_t\Phi_{int} \tag{1}$$

where Φ_{int} defines fixed costs of production, u_t^{int} is the degree of capacity utilization, K_t^{int} is the capital stock, h_t^{int} is the labor bundle:

$$h_t^{int} = \left\{ \left(\frac{1}{s}\right)^{\frac{\lambda_t^w}{1+\lambda_t^w}} \int_0^s \left[h_t^j\left(int\right)\right]^{\frac{1}{1+\lambda_t^w}} dj \right\}^{1+\lambda_t^w} \tag{2}$$

Firm int demand for labor type j is

$$h_t^j\left(int\right) = \left(\frac{W_t^j}{W_t}\right)^{-\frac{1+\lambda_t^w}{\lambda_t^w}} h_t^d \tag{3}$$

[1]See Juillard et al. (2008) and Dees et al. (2014) for a discussion of the difficulties associated to estimating multi-country DSGE models of the size required here.

[2]Kaplan et al. (2014) present a complementary interpretation of non-Ricardian behavior, allowing for the possibility that θ is composed by "poor hand-to-mouth" consumers, who do not hold any type of assets, and wealthy consumers who cannot smooth consumption over the busines cycle because their wealth is cocentrated in illiquid assets, such as housing.

where W_t^j is type j nominal wage, W_t is the aggregate nominal wage index and h_t^d is the aggregate labor demand. Each household supplies the labor bundle that firms demand. In each labor market j, wage-setting decisions are delegated to a monopolistic union and households supply the amount of labor that firms demand at W_t^j.[3] The time-varying parameter λ_t^w allows to incorporate wage markup shocks:

$$\log\left(\lambda_t^w\right) = (1-\rho_w)\log\left(\lambda^w\right) + \rho_w \log\left(\lambda_{t-1}^w\right) + \eta_t^w; \eta_t^w \sim N\left(0, \sigma_w^2\right). \tag{4}$$

$\varepsilon_t^{a,int}$ is a temporary technology shock, such that

$$\log\left(\varepsilon_t^{a,int}\right) = (1-\rho_{int})\log\left(\varepsilon^{a,int}\right) + \rho_{int}\log\left(\varepsilon_{t-1}^{a,int}\right) + \eta_t^{int} \tag{5}$$

and $z_t = z_{t-1}g_{z,t}$ is a labour-augmenting non-stationary technology shifter where

$$\log\left(g_{z,t}\right) = \left(1-\rho_{g_z}\right)\log\left(g_z\right) + \rho_{g_z}\log\left(g_{z,t-1}\right) + \eta_t^{g_z} \tag{6}$$

allows to incorporate technology shocks with a permanent effect on the level of productivity.

Intermediate firms operate in the domestic tradable and non-tradable sectors, T and N respectively. They face downward sloping demand curves obtained from standard consumption bundles

$$Y_t^X = \left[\left(\frac{1}{s}\right)^{\frac{\lambda_t^{p,X}}{1+\lambda_t^{p,X}}} \int_0^s Q_t^X(x)^{\frac{1}{1+\lambda_t^{p,X}}}\, dx\right]^{1+\lambda_t^{p,X}} \qquad X = T, N,$$

where markup shocks, i.e. shocks to the elasticity of substitution across goods are assumed to follow an AR(1) process with i.i.d. Normal error term:

$$\log\left(\lambda_t^{p,X}\right) = \left(1-\rho_{p,X}\right)\log\left(\lambda_t^{p,X}\right) + \rho_{p,X}\log\left(\lambda_{t-1}^{p,X}\right) + \eta_t^{p,X} \tag{7}$$

The final consumption bundle is:[4]

$$C_t = \left[\gamma_c^{\frac{1}{e}}\left(C_t^T\right)^{\frac{e-1}{e}} + (1-\gamma_c)^{\frac{1}{e}}\left(C_t^N\right)^{\frac{e-1}{e}}\right]^{\frac{e}{e-1}}; e > 1 \tag{8}$$

where C_t^T is defined as:

$$C_t^T = \left[\varpi^{\frac{1}{\upsilon}}\left(C_t^H\right)^{\frac{\upsilon-1}{\upsilon}} + (1-\varpi)^{\frac{1}{\upsilon}}\left(C_t^F\right)^{\frac{\upsilon-1}{\upsilon}}\right]^{\frac{\upsilon}{\upsilon-1}}; \upsilon > 1$$

Tradables incorporate domestic C_t^H and imported C_t^F tradable intermediate goods as inputs.

Household's i preferences $\varepsilon_t^c U_t^i\left(c_t^i, c_{t-1}, h_t^i\right)$ are characterized by non separability between consumption and labor effort and by consumption habits (Smets and Wouters, 2005, 2007; Albonico et al., 2016). To ensure that the model has a balanced growth path, consumption variables, $c_t^i = \frac{C_t^i}{z_t}$ and $c_t = \frac{C_t}{z_t}$, are normalized with the technology shifter z_t. Term ε_t^c is a preference shock affecting the subjective discount factor and evolving according to:

$$\log\left(\varepsilon_t^c\right) = (1-\rho_c)\log\left(\varepsilon^c\right) + \rho_c\log\left(\varepsilon_{t-1}^c\right) + \eta_t^c; \eta_t^c \sim N\left(0, \sigma_c^2\right) \tag{9}$$

[3] Following Galì et al. (2007) we assume that the fractions of Ricardian and non-Ricardian households is uniformly distributed across worker types. Since wage-setting decisions are centralized, this implies that households supply an identical amount of labor services in each labor market j.

[4] We postulate similar bundles for investment goods, see the Appendix for details.

Ricardian households allocate their resources between consumption C_t^o, investment in physical capital I_t^o, in public bonds B_{t+1}^othat pay the nominally riskless rate R_t^{ECB} and in a portfolio of state-contingent securities, A_t, that allow Ricardian households in the two regions to engage in mutual risk sharing. Their budget constraint is:

$$\begin{aligned}(1+\tau^c) P_{C,t} C_t^o + P_{I,t} I_t^o + A_t + B_{t+1}^o &= A_{t-1} + R_{t-1}^{ECB} B_t^o + \left(1-\tau^l-\tau^{wh}\right) W_t h_t^o + D_t^o \\ &+ \left(1-\tau^k\right)\left[\frac{R_t^k}{\varepsilon_{t-1}^b} u_t^o - a\left(u_t^o\right) P_{I,t}\right] K_t^o + \tau^k \delta P_{I,t} K_t^o + T_t^o\end{aligned} \tag{10}$$

where τ^c, τ^l, τ^{wh}, τ^k, T^o, respectively denote consumption, labor and capital income tax rates, social contributions levied on labor incomes, and lump-sum taxes: $P_{C,t}$ and $P_{I,t}$ are the price indexes for consumption and investment goods bundles; R_t^k is the rental rate of capital, $a\left(u_t^o\right)$ defines variable capacity utilization costs, and ε_t^b is a risk premium shock that creates a wedge between the return on capital accruing to the households and the price of capital paid by firms.[5]

$$\log\left(\varepsilon_t^b\right) = (1-\rho_b)\log\left(\varepsilon^b\right) + \rho_b \log\left(\varepsilon_{t-1}^b\right) + \eta_t^b; \eta_t^b \sim N\left(0, \sigma_b^2\right)$$

Physical capital accumulates as follows:

$$K_{t+1}^o = (1-\delta) K_t^o + \varepsilon_t^i \left[1 - S\left(\frac{I_t^o}{I_{t-1}^o}\right)\right] I_t^o$$

where δ is the depreciation rate and ε_t^i denotes an investment-specific technology shock:

$$\log\left(\varepsilon_t^i\right) = (1-\rho_i)\log\left(\varepsilon^i\right) + \rho_i \log\left(\varepsilon_{t-1}^i\right) + \eta_t^i; \eta_t^i \sim N\left(0, \sigma_i^2\right)$$

The term $S\left(\frac{I_t^o}{I_{t-1}^o}\right)$ represents standard investment adjustment costs.

Public consumption is exogenous and stochastic:

$$\log\left(\frac{g_t - g}{y}\right) = \rho_G \log\left(\frac{g_{t-1} - g}{y}\right) + \eta_t^G; \eta_t^G \sim N\left(0, \sigma_G^2\right) \tag{11}$$

where lower case letters stand for variables adjusted for growth, i.e. $g_t = G_t / z_t$, and g, y define steady state values.

As in Christoffel et al. (2008), the common monetary authority sets the nominal interest rate according to the following log-linear Taylor rule:

$$\begin{aligned}\hat{R}_t^{ECB} &= \phi_R \hat{R}_{t-1}^{ECB} + (1-\phi_R)\left(\phi_\pi \hat{\pi}_{t-1}^{EA} + \phi_y \hat{y}_t^{EA}\right) \\ &+ \phi_{\Delta\pi}\left(\hat{\pi}_t^{EA} - \hat{\pi}_{t-1}^{EA}\right) + \phi_{\Delta y}\left(\hat{y}_t^{EA} - \hat{y}_{t-1}^{EA}\right) + \hat{\varepsilon}_t^r\end{aligned} \tag{12}$$

where '^' denotes log deviations from steady state. $\pi_t^{EA} = \pi_{C,t}\left(\pi_{C,t}^*\right)^{1-s}$ is the Euro Area gross inflation rate and $y_t^{EA} = s y_t + (1-s) y_t^*$ is the Euro Area aggregate output. $\hat{\pi}_t^{EA}$ defines the deviation of inflation from steady state or target inflation.

[5] A similar kind of shock is introduced in Ratto et al. (2008) and Amano and Shukayev (2012).

2.1 Estimation strategy

The model is log-linearized around its steady state and then estimated using Bayesian techniques.

We adopt a two-stage approach. In the first stage we estimate a closed economy model in order to obtain estimates for the deterministic productivity trend and for the parameters of the Central Bank policy rule, including the inflation objective.[6] All these estimated values are then imposed in the second stage, when we estimate the four two-country models.[7]

The data sample is 1996Q2-2013Q3, due to data availability and to the difficulty of estimating the model after 2013Q3, when monetary policy was de facto constrained by the zero lower bound.

2.1.1 First-stage estimates

For the first-stage estimates we use the Euro area short-term nominal interest rate and 7 variables referred to the Euro area: real GDP, real private consumption, consumer price inflation (log difference in the overall HICP index), real investments, real compensation per employee, total employment, government spending.

Measurement equations are introduced to ensure that the observable variables are consistent with the properties of the model's balanced-growth path and with the underlying assumption that all relative prices are stationary. Output, consumption, investments, wages and government spending are transformed in log differences, thus approximating their growth rates. Following Christoffel et al. (2008), total employment has been detrended with a linear trend, which we define as $g_e * t$. The following measurement equation then incorporates the filtered variable $\ln e_t - g_e * t$:

$$\ln e_t - g_e * t = \hat{e}_t + \overline{e}$$

where $\overline{e}$ is an estimated constant. The auxiliary equation

$$\hat{e}_t = \frac{\beta}{1+\beta} E_t \hat{e}_{t+1} + \frac{1}{1+\beta} \hat{e}_{t-1} + \frac{(1-\xi_e)(1-\beta\xi_e)}{(1+\beta)\xi_e} \left(\hat{h}_t - \hat{e}_t\right) \tag{13}$$

relates the employment variable, $\hat{e}_t$, to the unobserved hours worked variable, $\hat{h}_t$. Parameter ξ_e determines the sensitivity of employment with respect to worked hours. For consumer price inflation the observation equation is:

$$\Delta \ln P_t = \pi_t + \overline{\overline{\pi}} \tag{14}$$

where $\overline{\overline{\pi}}$ is the estimated quarterly steady-state inflation rate.

The nominal interest rate is defined as:

$$\ln R_t^{ECB} = \hat{R}_t^{ECB} + \bar{R} \tag{15}$$

with $\bar{R}$ corresponding to the steady state nominal interest rate. For the remaining, non stationary variables we assume the following measurement equation:

$$\Delta \ln Y_t = \hat{y}_t - \hat{y}_{t-1} + \overline{\gamma} + \hat{g}_{z,t} + g_e \tag{16}$$

[6] Essentially the closed economy model is obatined by raising the share of the domestic economy to 1, so that the consumption and domestically produced bundles coincide, there is no distinction between traded and non-traded goods, any distinction between domestic and foreign residents falls.

[7] When we estimate the monetary policy parameters and the productivity growth rate in each model, we obtain slightly different results for these variables, but our conclusions concerning cross-country differences are confirmed. Results are available upon request.

where $\overline{\gamma} = 100(g_z - 1)$ and $\hat{g}_{z,t}$ respectively denote the estimated deterministic and stochastic growth trend components.

This closed-economy model is estimated assuming interest rate, risk premium, investment-specific, price and wage markup, government spending, temporary and permanent productivity shocks.

2.1.2 Second-stage estimates

In the second stage we use 9 time series characterizing the specific domestic country, that is, real GDP, real private consumption, consumer price inflation (log difference in the overall HICP index), real investments, real compensation per employee, total employment, government spending, non-tradables inflation (log difference in the services HICP index, as in Kolasa, 2009) and nontradables GDP (proxied by services GDP, as in Rabanal, 2009). Nine additional observables symmetrically define observables in the rest of the Euro area.

For each model, we estimate the employment trend coefficients g_e and g_e^* with simple OLS methods. Then, the employment measurement equations are defined as:

$$\begin{aligned} \ln e_t - g_e * t &= \hat{e}_t + \overline{e} \\ \ln e_t^* - g_e^* * t &= \hat{e}_t^* + \overline{e}^* \end{aligned}$$

where

$$\hat{e}_t = \frac{\beta}{1+\beta} E_t \hat{e}_{t+1} + \frac{1}{1+\beta} \hat{e}_{t-1} + \frac{(1-\xi_e)(1-\beta\xi_e)}{(1+\beta)\xi_e} \left(\hat{h}_t - \hat{e}_t\right) \tag{17}$$

$$\hat{e}_t^* = \frac{\beta}{1+\beta} E_t \hat{e}_{t+1}^* + \frac{1}{1+\beta} \hat{e}_{t-1}^* + \frac{(1-\xi_e^*)(1-\beta\xi_e^*)}{(1+\beta)\xi_e^*} \left(\hat{h}_t^* - \hat{e}_t^*\right) \tag{18}$$

As shown above, for each of the 4 models we consider an interest rate shock and, a set of country-specific shocks: two transitory sectoral TFP shocks, one shock to the productivity trend, a risk premium shock, an investment-specific shock, a preference shock, a government spending shock, price and wage markup shocks.

For sectoral inflation variables, the observation equations are:

$$\Delta \ln P_t = \pi_t + \overline{\overline{\pi}} \tag{19}$$

$$\Delta \ln P_t^* = \pi_t^* + \overline{\overline{\pi}} \tag{20}$$

where $\overline{\overline{\pi}}$ is set according to the first-stage estimates.

For the remaining non stationary variables the measurement equations are:[8]

$$\Delta \ln Y_t = \hat{y}_t - \hat{y}_{t-1} + \overline{\gamma} + \hat{g}_{z,t} + g_e \tag{21}$$

$$\Delta \ln Y_t^* = \hat{y}_t^* - \hat{y}_{t-1}^* + \overline{\gamma} + \hat{g}_{z,t}^* + g_e^* \tag{22}$$

were $\overline{\gamma}$ is retrieved from first-stage estimates. For government expenditures we impose the following measurement equations:

[8] We allow for a measurement error in nontradables GDP equations.

$$\Delta \ln G_t = \frac{y}{g}\left(\hat{g}_t - \hat{g}_{t-1}\right) + \overline{\gamma} + \hat{g}_{z,t} + g_e \quad (23)$$

$$\Delta \ln G_t^* = \frac{y^*}{g^*}\left(\hat{g}_t^* - \hat{g}_{t-1}^*\right) + \overline{\gamma} + \hat{g}_{z,t}^* + g_e^* \quad (24)$$

where $\hat{g}_t$ is defined as $\frac{g_t - g}{y}$.

2.1.3 Calibration and priors

We calibrate a number of parameters at the same level for all EMU countries. Following Christoffel et al. (2008), the discount factor β is fixed at 0.9988, the steady-state depreciation rate δ is 0.025, the capital shares α_{int} are set at 0.3, the steady state net price and wage markups are fixed at 35% and 30% respectively, redistributive transfers are assumed to determine a steady state consumption ratio $c^{rt}/c^{o} = 0.8$.

Another set of parameters are calibrated using average sample data. For each of the four countries in Table 1 we report the country-size parameters s, the goods shares $(\gamma_c, \gamma_c^*, \gamma_i, \gamma_i^*, \varpi, \varpi^*)$, the constant tax and social contributions rates $\left(\tau^c, \tau^{c,*}\tau^l, \tau^{l,*}, \tau^k, \tau^{k,*}, \tau^l, \tau^{wf}\right)$, the steady state public-consumption- and debt-to-GDP ratios $\left(\frac{g}{y}, \frac{g^*}{y^*}, \frac{b}{y}, \frac{b^*}{y^*}\right)$.

Parameters s correspond to the HICP weights for each of the 4 countries. The shares of tradable consumption goods (γ_c, γ_c^*) correspond to sample-average shares of goods in the HICP basket. The shares of investment goods (γ_i, γ_i^*) are measured by the share of non-construction investments over total investment expenditures. The share of home produced goods in the tradable index ϖ is equal to one minus the average sample-period ratio between total bilateral imports and GDP. The rest of the Euro Area counterpart, ϖ^*, is obtained endogenously through the steady state. The constant tax rates are average sample ratios between the relevant revenue and tax-base series.[9] We use average sample ratios for calculating government-spending-to-GDP and debt-to-GDP ratios.[10]

Parameters g_e and g_e^* are obtained from OLS estimates of $\ln e_t$. For each country and group of countries we estimate the following equation:

$$\ln e_t = const + g_e * t + \varepsilon_t$$

We impose these values in our calibration (see Table 1). Note that for Spain we obtain a value (0.53) which is dramatically higher than other countries'.

The remaining parameters are estimated with Bayesian techniques. Priors, reported in Table 2, are set in line with the literature on Euro Area models (see Smets and Wouters 2003, 2005; Christoffel et al. 2008; Rabanal 2009; Kolasa 2009). All the parameters priors are set symmetrically. In particular, parameters measuring the persistence of the shocks are assumed to be Beta distributed, with mean 0.5 and standard deviation 0.1 and the standard errors of the innovations are assumed to follow Gamma distributions, similarly to Rabanal (2009). The parameters governing price and wage setting, habits, utilization elasticity, interest rate smoothing and the steady state fraction of LAMP are also Beta distributed. The fractions of LAMP θ, θ^* are assumed to be Beta distributed with mean 0.4 and standard deviation 0.1, in line with the recent results obtained

[9] As a proxy for employees and employers social security contributions we assume that 1/3 of contributions are paid by the households while 2/3 of contributions are paid by firms, as in Christoffel et al. (2008).

[10] We derive the difference between aggregate transfers and taxes to GDP ratios as a residual from the steady state government budget constraint.

for the Euro Area by Albonico et al. (2014). The priors for the elasticity of substitutions in the consumption indices (e, υ) are set in line with Rabanal (2009) as Normal(1, 0.5) distributions.

In the closed economy model, we estimate the monetary authority's long-run (net) quarterly inflation objective $100\,(\bar{\pi}-1)$ assuming a prior mean of 0.5% (2% in annual terms), consistent with the ECB's quantitative definition of price stability. The trend growth rate of the economy is estimated with a Normal distribution with mean 0.6 (corresponding to 2.4% in annual terms) and 0.1 standard deviation.

Risk aversion, the inverse of Frisch elasticity and the parameters of the Taylor rule are Normally distributed, whereas the parameter defining investment adjustment costs is Gamma distributed.[11]

Table 1: Calibration of parameters

Parameters definition		***DE***	***FR***	***IT***	***ES***
β	discout factor	0.998			
δ, δ^*	depreciation rate	0.025			
α_N, α_H, $\alpha_{N,*}$ α_F	capital shares	0.3			
λ_p, λ_p^*	price markup	0.35			
λ_w, λ_w^*	wage markup	0.3			
$\frac{c^{rt}}{c^o}$, $\frac{c^{rt,*}}{c^{o,*}}$	consumption ratio	0.8			
s	country's size	0.294	0.205	0.183	0.109
γ_c	domestic share of tradable goods in consumption basket	0.624	0.611	0.616	0.644
γ_c^*	REA share of tradable goods in consumption basket	0.597	0.605	0.604	0.602
γ_i	domestic share of tradable goods in investment basket	0.43	0.460	0.48	0.37
γ_i^*	REA share of tradable goods in investment basket	0.49	0.472	0.47	0.48
ϖ	fraction of domestic-produced goods in the tradable index	0.91	0.958	0.897	0.902
τ_c	domestic consumption tax rate	0.215	0.277	0.232	0.187
τ_c^*	REA consumption tax rate	0.23	0.211	0.223	0.229
τ_l	domestic wage tax rate	0.303	0.226	0.402	0.199
τ_l^*	REA wage tax rate	0.229	0.262	0.222	0.262
τ_k	domestic capital tax rate	0.206	0.178	0.212	0.185
τ_k^*	REA capital tax rate	0.159	0.175	0.168	0.175
τ_{wh}	domestic social contribution tax rate in payroll	0.127	0.142	0.149	0.108
τ_{wh}^*	REA social contribution tax rate in payroll	0.131	0.126	0.125	0.132
τ_{wf}	domestic social contribution charged on employer	0.254	0.285	0.298	0.216
τ_{wf}^*	REA social contribution charged on employer	0.262	0.253	0.251	0.265
$\frac{b}{y}$	domestic debt to output ratio	0.912*4	0.687*4	1.091*4	0.569*4
$\frac{b^*}{y^*}$	REA debt to output ratio	0.669*4	0.772*4	0.679*4	0.777*4
$\frac{g}{y}$	domestic government to output ratio	0.186	0.229	0.189	0.180
$\frac{g^*}{y^*}$	REA government to output ratio	0.207	0.192	0.202	0.203
g_e	domestic employment trend	0.139	0.242	0.214	0.532
g_e^*	REA employment trend	0.257	0.220	0.226	0.181

[11] Where applicable, parameter calbrations and prior distributions are adopted in the estimated euro area-wide model.

Table 2: Prior distributions of parameters

Parameters definition		shape	mean	std dev
e, υ	elasticity of substitutions in the consumption indices	gamma	1	0.5
σ, σ^*	intertemporal elastiticity of substitutions	norm	2.5	0.375
ζ, ζ^*	degree of external habit in consumption	beta	0.6	0.1
ϕ_l, ϕ_l^*	inverse of Frisch elestiticity	norm	2.5	0.4
θ, θ^*	fraction of LAMP	beta	0.4	0.1
γ_I, γ_I^*	investment adjustment costs	gamma	4	0.5
σ_u, σ_u^*	capital utilization parameter	beta	0.5	0.15
χ_p^N, $\chi_p^{N,*}$, χ_p^H, χ_p^F	price indexation to past inflation	beta	0.75	0.1
ξ_p^N, $\xi_p^{N,*}$, ξ_p^H, ξ_p^F	price rigidity	beta	0.75	0.1
χ_w, χ_w^*	wage indexation to past inflation	beta	0.75	0.1
ξ_w, ξ_w^*	wage rigidity	beta	0.75	0.1
ξ_e, ξ_e^*	Calvo employment	beta	0.5	0.15
ρ	autoregressive coefficient of shocks	beta	0.5	0.1
σ^{gz}, $\sigma^{gz,*}$, $\sigma^{a,N}$, $\sigma^{a,H}$, $\sigma^{a,N,*}$, $\sigma^{a,F}$	standard deviation shocks	gamma	0.7	0.3
σ^b, $\sigma^{b,*}$, σ^r, σ^π, σ^{rsh}	standard deviation shocks	gamma	0.4	0.2
σ^c, $\sigma^{c,*}$, σ^i, $\sigma^{i,*}$, σ^w, $\sigma^{w,*}$, σ^p, $\sigma^{p,*}$, σ^g, $\sigma^{g,*}$	standard deviation shocks	gamma	1	0.5

3 Results

Our estimates of the policy rule parameters and of the productivity growth trend are reported in Table 3

Table 3: Posterior mean estimates of parameters and shocks

Parameters		Prior distribution			Posterior	
		Shape	Mean	std dev.	Mean	90% HD
$\bar{\gamma}$	Deterministic trend	Norm	0.60	0.10	0.201	(0.11; 0.29)
$\bar{\pi_*}$	Quarterly inflation objective	Norm	0.50	0.10	0.623	(0.50; 0.75)
ϕ_r	Lagged interest rate response	Beta	0.90	0.05	0.835	(0.80; 0.87)
ϕ_π	Interest rate response to inflation	Norm	1.70	0.10	1.743	(1.59; 1.90)
ϕ_y	Interest rate resp. to output	Norm	0.12	0.05	0.006	(0.00; 0.02)
$\phi_{\Delta y}$	Interest rate resp. to change in output	Norm	0.06	0.05	0.133	(0.11; 0.16)
$\phi_{\Delta \pi}$	Interest rate resp. to change in inflation	Norm	0.30	0.10	0.243	(0.18; 0.30)
LDD						-240.086

Tables 4 and 5 show the country-specific posterior mean estimates of structural parameters and shocks obtained in the 4 second-stage estimates.[12]

We obtain fairly similar posterior estimates for the four countries, which are well identified. We observe important cross-country differences in the fraction of LAMP consumers, which is particularly large in Germany and small in Spain.[13]

The elasticities of substitution e and v have in general low posterior means. These low values are quite common in the literature that estimates open economy sticky price models because low elasticities are needed to explain higher volatility of relative prices than relative quantities (see Lubik and Schorfheide 2005, Rabanal and Tuesta 2006). Rabanal (2009) finds a similar estimates for these parameters in Spain.

3.1 Variance decompositions

The output variance decomposition (Table 6) highlights the different role played by supply and demand shocks in the 4 countries. Demand shocks played a relatively limited role in the souther countries. In Italy technology shocks and, to a lesser extent, public consumption shocks explain a relatively larger fraction of output growth volatility. Domestic (price) markup shocks were particularly important in Germany and Spain.

Turning to the inflation variance decomposition (Table 7), we see that a combination of technology and mark up shocks explains the bulk of inflation volatility in all countries.

Finally, volatility of the public consumption ratio is explained by non-policy shocks in all countries but Italy, where fiscal discretion was the main source of volatility (Table 8).

All in all, the variance decompositions emphasize the limited role played by demand shocks in explaining the volatility of GDP growth and inflation in southern countries.

[12] Visual diagnostics of the estimation results are available upon request. The posterior distributions are computed considering 4 Monte Carlo Markov chains of 250,000 draws each, with 20% draws being discarded as burn-in draws. The average acceptance rate is comprised between 28 and 34 percent.

[13] Kaplan et al. (2014) estimate a similar ranking for the fraction of illiquid households, but cross-country differences are relatively smaller.

Table 4: Posterior mean estimates of parameters

	Parameters	DE	FR	IT	ES
e	Elasticity of subs. in the consumption index	0.194	0.175	0.347	0.222
		(0.056-0.325)	(0.047-0.295)	(0.127-0.552)	(0.064-0.372)
υ	Elasticity of subs. in the tradable consump. index	0.475	0.428	1.766	0.173
		(0.105-0.843)	(0.099-0.735)	(0.723-2.756)	(0.043-0.301)
σ	Intert. elasticity of subs. in the consumption indices	2.508	2.678	2.441	1.880
		(2.013-3.001)	(2.117-3.222)	(1.942-2.952)	(1.444-2.350)
b	Degree of external habit formation	0.492	0.606	0.573	0.479
		(0.307-0.666)	(0.438-0.766)	(0.408-0.737)	(0.297-0.651)
ϕ_l	Inverse Frisch labor elasticity	2.297	2.451	2.717	2.563
		(1.686-2.897)	(1.838-3.080)	(2.078-3.354)	(1.853-3.212)
θ	Fraction of LAMP	0.465	0.369	0.465	0.178
		(0.349-0.588)	(0.219-0.519)	(0.339-0.593)	(0.086-0.269)
γ_I	Investment adjustment costs	5.002	5.309	4.555	4.986
		(3.986-5.979)	(4.420-6.245)	(3.720-5.419)	(4.235-5.661)
σ_u	Capital utilization	0.361	0.307	0.423	0.262
		(0.288-0.435)	(0.222-0.398)	(0.350-0.504)	(0.168-0.346)
χ_p^N	Price index. to past inflation in non-trad. goods	0.681	0.880	0.868	0.882
		(0.505-0.855)	(0.797-0.966)	(0.771-0.967)	(0.804-0.964)
χ_p^H	Price index. to past inflation in tradable goods	0.340	0.365	0.270	0.309
		(0.203-0.468)	(0.226-0.499)	(0.151-0.381)	(0.183-0.427)
χ_p^F	Price index. to past inflation in imports	0.411	0.811	0.370	0.494
		(0.249-0.572)	(0.689-0.931)	(0.224-0.521)	(0.336-0.668)
ξ_p^N	Price stickiness in non-tradable goods	0.883	0.896	0.889	0.895
		(0.862-0.900)	(0.890-0.900)	(0.876-0.900)	(0.889-0.900)
ξ_p^H	Price stickiness in tradable goods	0.558	0.540	0.691	0.582
		(0.509-0.602)	(0.500-0.572)	(0.644-0.739)	(0.529-0.637)
ξ_p^F	Price stickiness in imports	0.565	0.637	0.436	0.550
		(0.510-0.621)	(0.587-0.690)	(0.387-0.472)	(0.496-0.606)
χ_w	Wage indexation to past inflation	0.712	0.743	0.787	0.757
		(0.537-0.910)	(0.595-0.891)	(0.640-0.937)	(0.622-0.895)
ξ_w	Wage stickiness	0.851	0.833	0.688	0.854
		(0.812-0.894)	(0.791-0.876)	(0.619-0.758)	(0.817-0.894)
ξ_e	Calvo employment	0.525	0.589	0.547	0.653
		(0.470-0.580)	(0.541-0.637)	(0.501-0.595)	(0.614-0.692)

3.2 Historical decompositions

In this section, we present the historical decompositions for the growth rates of output, exchange rate, real wages index and public-consumption-to-GDP ratio (Figures 4, 6 7 and 8 respectively). We address the key question whether exceedingly favorable demand conditions paved the way for the post 2010 crisis or whether the sovereign bond crisis should be seen as a game changer that created a new economic environment within the Eurozone.

3.2.1 Output growth

To begin our discussion it may be helpful to look at the historical decomposition for the whole Eurozone, obtained from first-stage estimates (Figure 3). Demand shocks, including monetary and fiscal policies, were crucial to determine the pre-financial crisis growth acceleration. The financial crisis was mainly determined by a sequence of large non-policy demand shocks. Then, the post 2010 downturn is explained by a combination of markup, technology and non-policy demand shocks.

In Germany a sequence of favorable demand shocks supported growth in the 2003-2007 period, then there was a huge reversal in domestic demand shocks at the time of the 2007-2008 global crisis. Finally, the 2010- period was characterized by a reduction in volatility. For France we observe a similar pattern in the 2003-2007 period, when the favorable demand shocks were crucial to support growth, but in the subsequent years technology shocks contributed more to the growth slowdown.

Results for Italy are quite different. First, in the 2003-2007 period we cannot detect a growth acceleration, and demand shocks remained subdued relative to Germany and France. Unlike these two countries, public consumption shocks in Italy played an important role, but there is no evidence of an expansionary bias. Second, since 2008 and particularly after 2010 adverse technology shocks became very important. Figure 5 presents the decomposition of technology shocks contributions: the post 2008 years are characterized by a sequence of adverse permanent shocks. Third, public consumption shocks were erratic and became almost irrelevant from 2010 onwards.

Output growth decomposition for Spain highlights some specific business cycle features. First, the pre-2008 high-growth period is characterized by a combination of favorable demand and technology shocks, where the latter were mainly characterized by permanent shocks. The global financial crisis period is marked by adverse demand shocks. Since 2010, the onset of the sovereign bond crisis is associated to a sequence of adverse permanent productivity shocks that were decisive to determine the growth slowdown. Relative to Italy, public expenditure shocks had a very limited influence on GDP growth, but after 2010 we observe an increase in their amplitude.

3.2.2 Real exchange rate growth

According to popular wisdom the early EMU years where characterized by exchange rate appreciations in the periphery. As a matter of fact, the Italian real exchange rate mainly depreciated prior to the crisis, and domestic demand conditions contributed to this outcome. Spain is the only country which was characterized by pre-financial crisis appreciation. Our decomposition highlights the overwhelming role played by technology shocks in determining this outcome.

3.2.3 Real wage growth

Demand driven wage increases have been singled out as the main culprit of competitiveness losses in the south of the Eurozone. In fact neither country was characterized by wage growth rates that systematically exceeded the corresponding wage growth rates for the rest of the Eurozone. Moreover, demand shocks played a limited role in determining wage dynamics in these two countries before the onset of the financial crisis. After 2010 technology shocks became the dominant force behind the observed slowdown in wage growth rates.

3.2.4 Public consumption ratios

As mentioned above, in Italy public consumption shocks were relatively more important up to 2010, but we cannot identify a tendency to implement undisciplined discretionary policies. After 2010 the public consumption ratio is remarkably more stable. Germany, Italy and Spain share a tendency to implement accommodative discretionary policies: shocks often drive the ratio in the same direction of non-policy shocks. In France the ratio is almost entirely determined by non-policy shocks.

3.2.5 Summing up

Our results cannot support the view that pre-2007 growth in the two southern countries was driven by a demand boom. Perhaps surprisingly, favorable demand conditions were relatively more important in France and in Germany. Competitiveness indicators, measured by real exchange rate and wage growth confirm this conclusion, as demand factors played a negligible role in determining these variables in Italy and Spain.

One striking result is that asymmetric shocks to the productivity trend play a non-negligible role in explaining the favorable performance of Spain before the crisis, and are major determinants of the turnaround in growth perspectives for the two southern countries after 2010.

Given the differential role of permanent technology shocks in the 4 countries, in Figure 9 for each country we plot the estimated productivity growth rates $\ln(g_{z,t})$ for the four countries. Results are striking. Productivity growth was below trend in Germany up until 2007 and became strongly positive since then. France experienced more favorable growth rates in the early EMU years and maintained a good performance after 2010. Evidence for Italy is particularly gloomy: productivity growth has fallen since 2002, and the situation worsened dramatically after 2010. Up until the 2008 financial crisis Spain experienced the fastest productivity growth rates, well above the balanced growth rate. Then productivity growth dramatically deteriorated after 2010.

One might wonder whether our results concerning the importance of permanent technology shocks might be due to model misspecification, possibly due to our choice to abstract from explicit modelling of financial frictions. To answer this question in Figure 10 we report the IRFs to a temporary shock to the productivity growth rate estimated for Spain. A temporary slowdown in productivity growth has a contractionary effect. Consumption is reduced, investment output and hours worked fall, along with the real wage and inflation. Relative to temporary contractionary shocks there are two key distinct features: the first is that Ricardian consumers now react to the permanent income reduction and their willingness to smooth consumption is therefore limited, the second is that the shock causes permanent adjustments in the long run. In fact neither effects could possibly materialize in DSGE models accounting for banking frictions, such as the ones estimated in Brzoza-Brzezina and Kolasa (2013).

To conclude our discussion, note that the deflationary impact of the shock determines a real depreciation both in the short and in the long run. This last result allows to provide an alternative interpretation of the persistent real exchange rate appreciation observed for Spain before the financial crisis, in sharp contrast with the view that interprets it as the consequence of a demand boom.

Table 5: Posterior mean estimates of shocks

Parameters		DE	FR	IT	ES
Shock persistence					
$\rho_{a,H}$	TFP of tradable shock	0.898	0.921	0.931	0.929
		(0.868-0.928)	(0.899;0.944)	(0.912-0.953)	(0.911-0.948)
$\rho_{a,N}$	TFP of Non-tradable shock	0.941	0.832	0.540	0.950
		(0.928-0.953)	(0.764;0.898)	(0.390-0.683)	(0.946-0.953)
ρ_c	Preference shock	0.489	0.850	0.790	0.801
		(0.326-0.650)	(0.787;0.918)	(0.675-0.906)	(0.686-0.916)
ρ_b	Risk premium shock	0.832	0.900	0.740	0.895
		(0.780-0.884)	(0.863;0.940)	(0.648-0.833)	(0.863-0.928)
ρ_i	Investment specific shock	0.598	0.603	0.811	0.494
		(0.474-0.719)	(0.456;0.738)	(0.713-0.912)	(0.319-0.664)
ρ_p	Price markup shock	0.531	0.834	0.740	0.854
		(0.379-0.674)	(0.772;0.902)	(0.636-0.843)	(0.799-0.908)
ρ_w	Wage markup shock	0.477	0.663	0.458	0.578
		(0.345-0.615)	(0.548;0.784)	(0.329-0.587)	(0.457-0.711)
ρ_g	Government shock	0.887	0.471	0.483	0.773
		(0.842-0.935)	(0.307;0.647)	(0.327-0.641)	(0.662-0.881)
ρ_{gz}	Permanent technology shock	0.400	0.546	0.893	0.693
		(0.281-0.499)	(0.434-0.697)	(0.867-0.920)	(0.544-0.849)
Standard deviation shock					
$\sigma_{a,H}$	TFP of tradable shock	1.516	1.609	1.614	2.273
		(1.268-1.779)	(1.363-1.858)	(1.346-1.875)	(1.910-2.656)
$\sigma_{a,N}$	TFP of Non-tradable shock	1.407	1.086	1.929	1.916
		(1.176-1.635)	(0.893-1.280)	(1.584;2.265)	(1.641-2.207)
σ_c	Preference shock	1.457	1.838	2.382	1.860
		(0.948-1.956)	(1.322-2.325)	(1.711;3.037)	(1.291-2.429)
σ_b	Risk premium shock	2.872	2.862	3.052	2.863
		(2.589-3.170)	(2.540-3.170)	(2.906;3.170)	(2.561-3.170)
σ_i	Investment specific shock	0.311	0.200	0.180	0.173
		(0.236-0.388)	(0.143-0.254)	(0.134;0.224)	(0.115-0.229)
σ_p	Price markup shock	0.122	0.030	0.060	0.069
		(0.087-0.155)	(0.022-0.038)	(0.042;0.078)	(0.052-0.086)
σ_w	Wage markup shock	0.250	0.133	0.756	0.375
		(0.181-0.315)	(0.096-0.170)	(0.598;0.911)	(0.287-0.462)
σ_g	Government shock	0.178	0.055	0.371	0.308
		(0.148-0.206)	(0.041-0.069)	(0.311;0.429)	(0.259-0.358)
σ_{gz}	Permanent technology shock	0.536	0.310	0.156	0.408
		(0.419-0.651)	(0.239-0.380)	(0.111;0.199)	(0.217-0.603)
σ_r	Monetary policy shock	0.094	0.119	0.129	0.091
		(0.079-0.108)	(0.100-0.138)	(0.110;0.147)	(0.076-0.104)
σ_{me}	Measurement error in gdp by non-tradable goods	0.639	0.461	0.792	0.659
		(0.562-0.714)	(0.401-0.522)	(0.710;0.871)	(0.578-0.738)
σ_e	Measurement error in employment	0.7924	-0.431	-0.489	0.380
		(-0.122–1.678)	(-1.247-0.366)	(-1.091-0.083)	(-1.261-2.006)
LDD		-1053	-1028	-1116	-1233

Table 6: Output variance decomposition

	DE	FR	IT	ES
Domestic Demand	35.94%	45.32%	24.26%	28.88%
Domestic Technology	53.66%	42.54%	46.08%	46.34%
Domestic Markup	4.69%	5.93%	5.35%	13.26%
Domestic Public consumption	2.37%	0.33%	21.15%	3.86%
Monetary policy	1.01%	1.75%	1.34%	0.93%
REA	2.33%	4.13%	1.81%	6.74%
Contribution of individual domestic shock				
TFP Non tradables	15.89%	11.60%	21.13%	17.01%
TFP tradables	13.62%	20.09%	12.96%	15.38%
Permanent technology	24.14%	10.85%	12.00%	13.95%
Price markups	1.66%	1.57%	1.38%	4.50%
Wage markups	3.03%	4.36%	3.97%	8.76%

Table 7: Inflation Variance decomposition

	DE	FR	IT	ES
Domestic Demand	9.22%	9.07%	12.14%	4.38%
Domestic TFP Non tradables	20.36%	6.34%	4.53%	14.97%
Domestic TFP tradables	46.88%	62.81%	36.18%	52.15%
Domestic permanent technology	3.68%	2.37%	20.32%	2.71%
Domestic Price markups	3.15%	0.61%	1.73%	0.86%
Domestic Wage markups	11.74%	10.29%	13.04%	20.98%
Domestic Public consumption	0.29%	0.00%	0.41%	0.10%
Monetary policy	1.19%	2.13%	4.17%	0.50%
REA	3.48%	6.38%	7.48%	3.35%

Table 8: Governement spending over GDP variance decomposition

	DE	FR	IT	ES
Non-policy	77.06%	96.56%	28.36%	68.67%
Monetary policy	1.06%	1.91%	0.55%	0.76%
Government spending	21.89%	1.53%	71.09%	30.58%

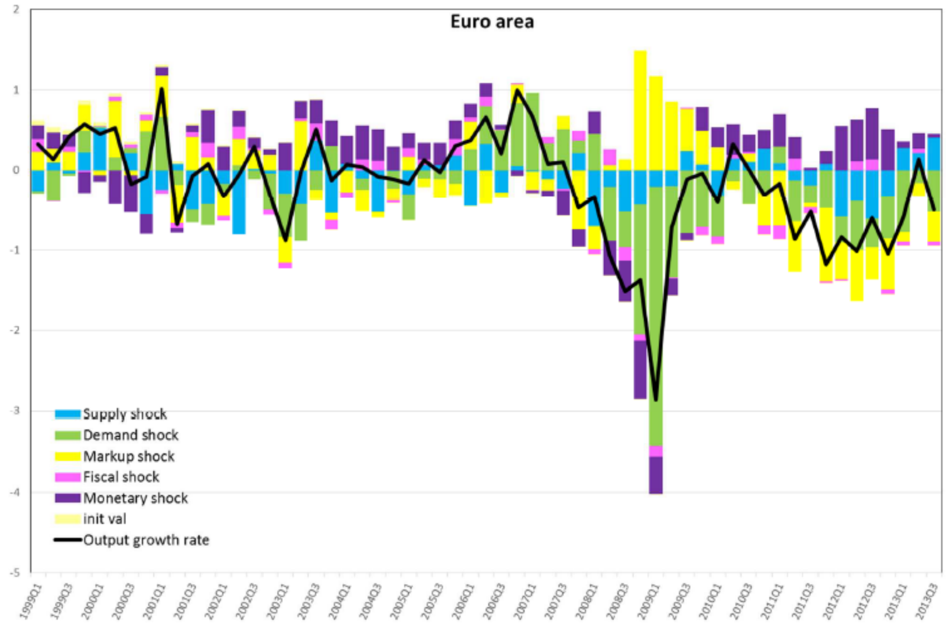

Figure 3: Historical decomposition of output growth in the Euro area.

Figure 4: Historical decomposition of output growth.

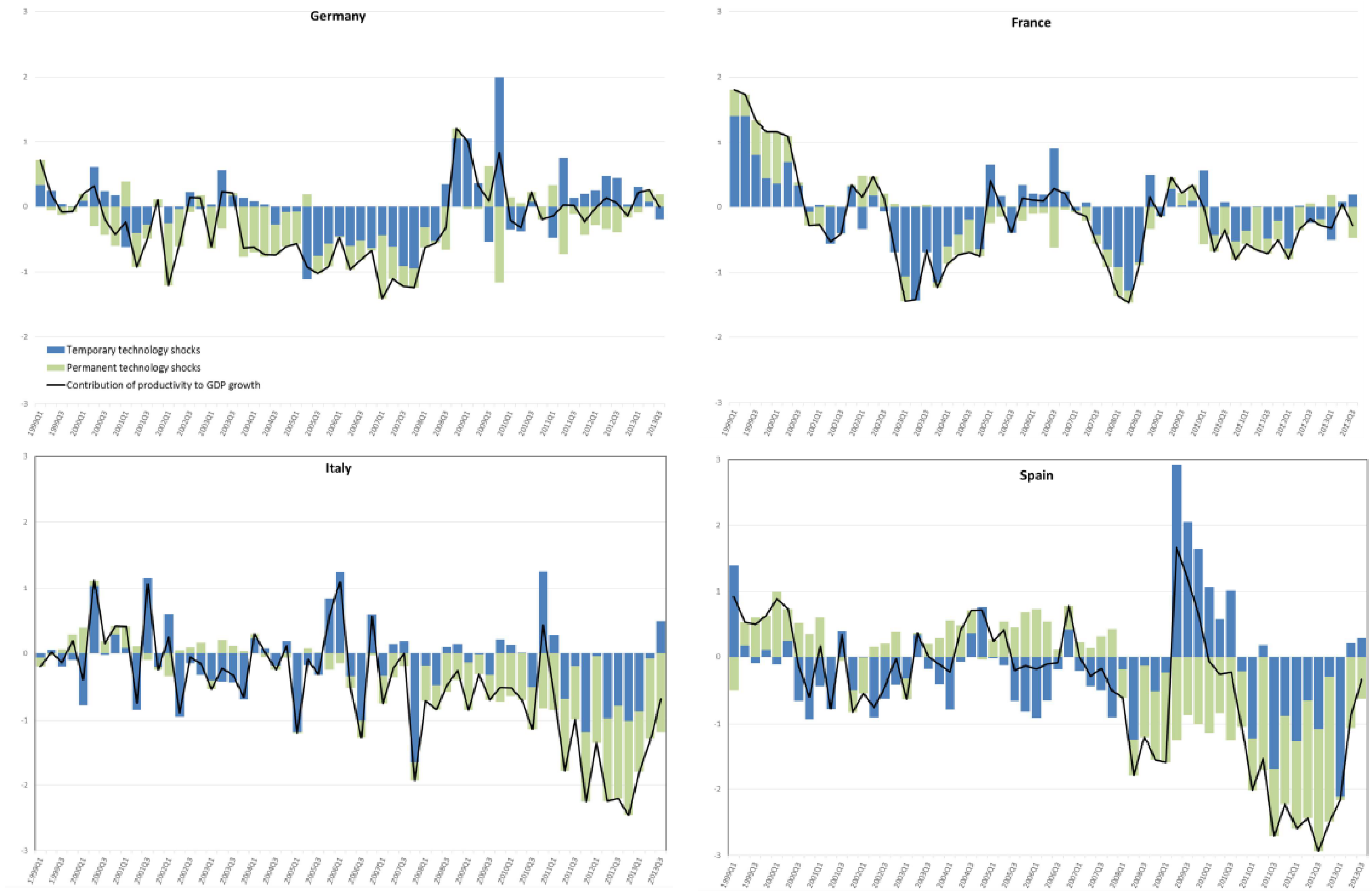

Figure 5: Contribution of productivity shocks to output growth.

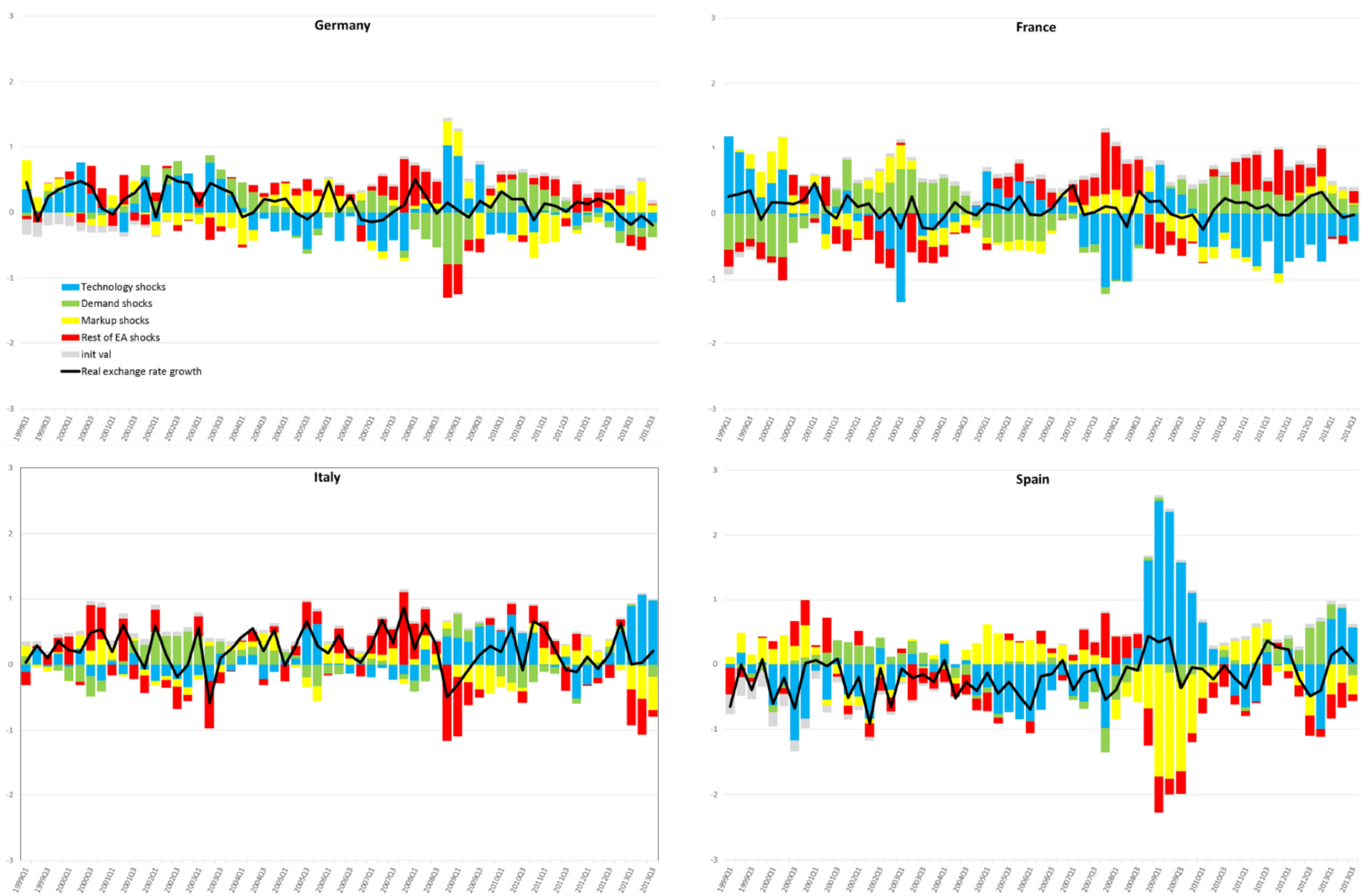

Figure 6: Historical decomposition of real exchange rate growth.

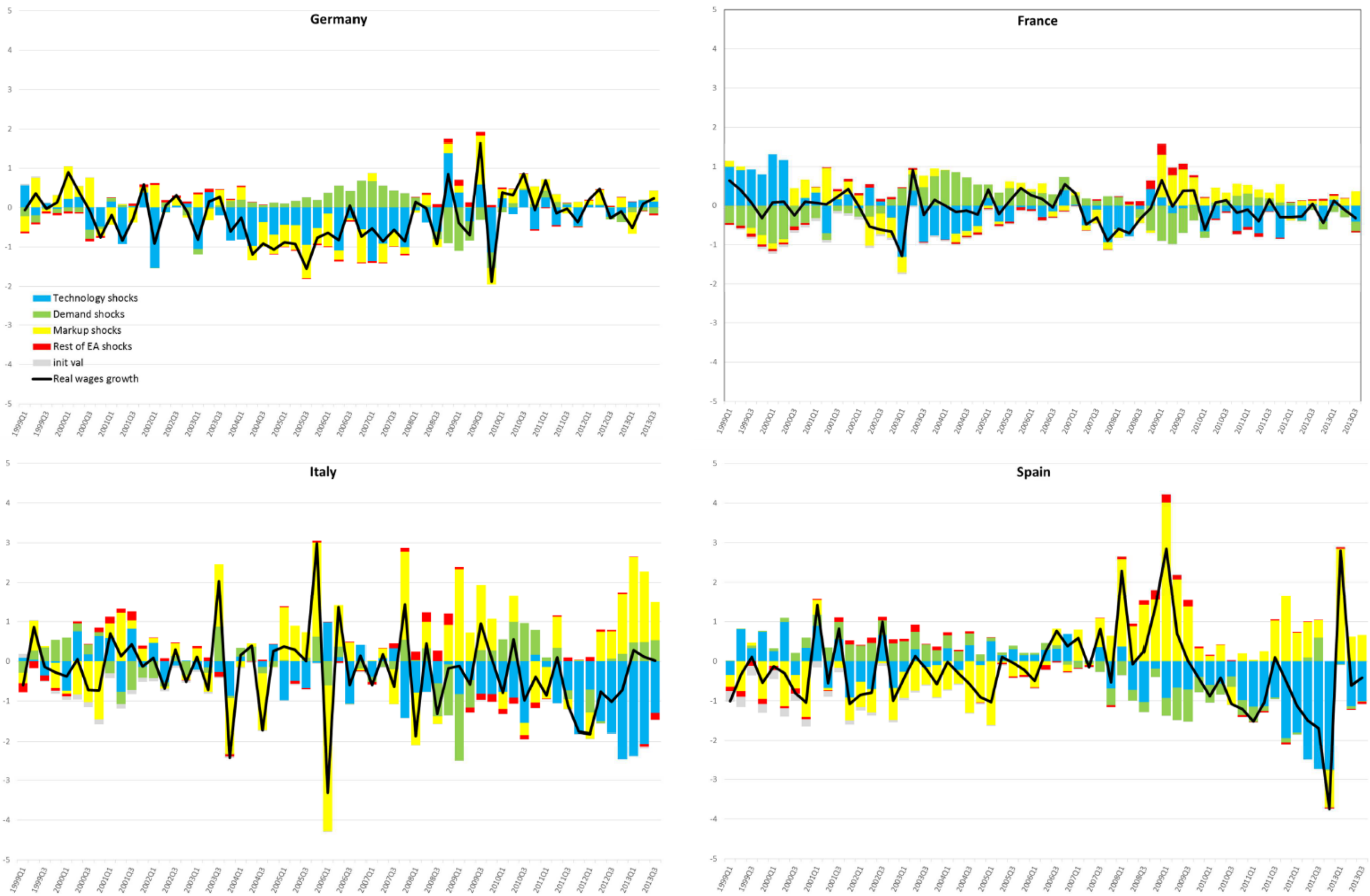

Figure 7: Historical decomposition of real wages growth.

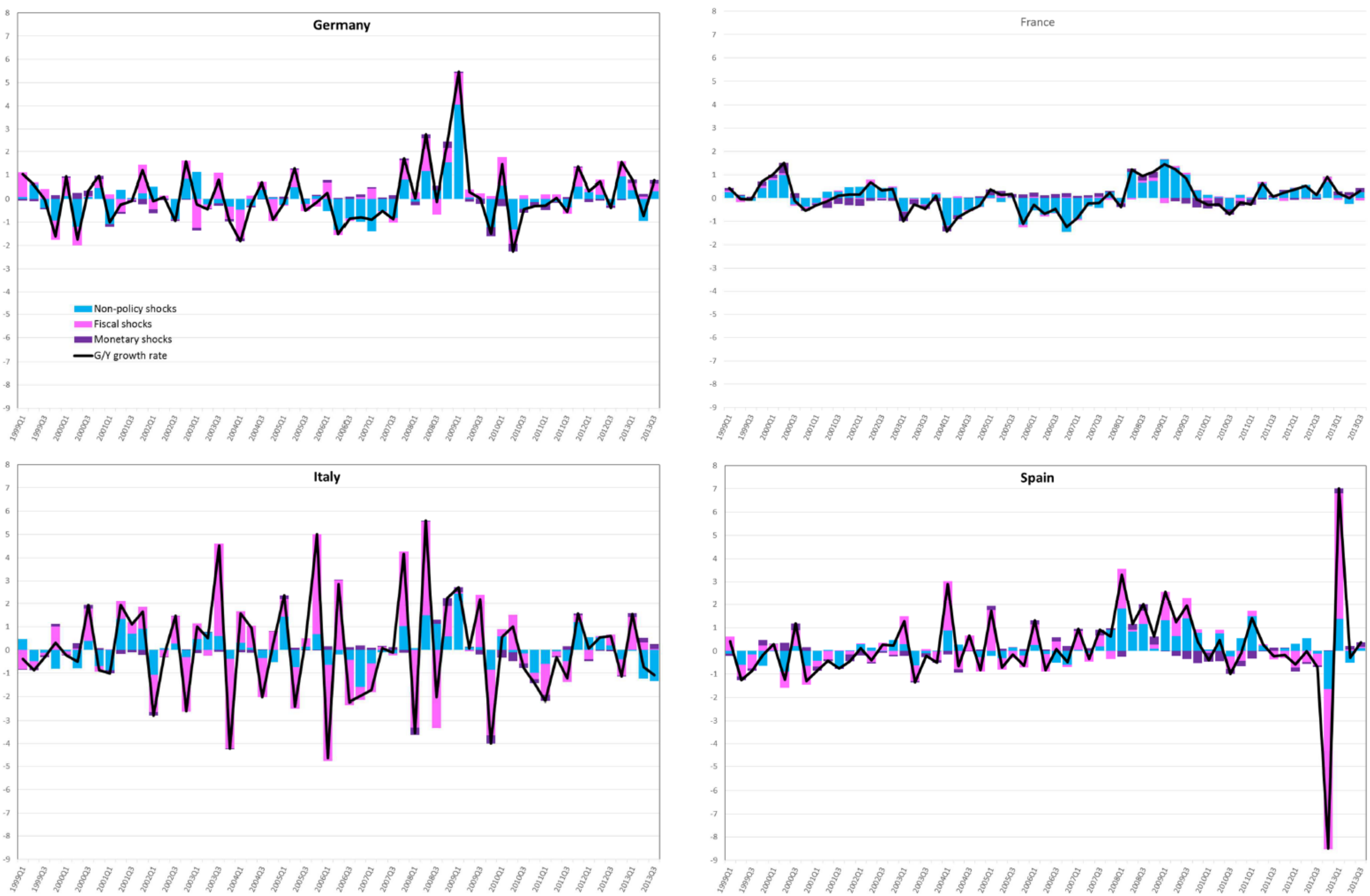

Figure 8: Historical decomposition of G/Y growth rate.

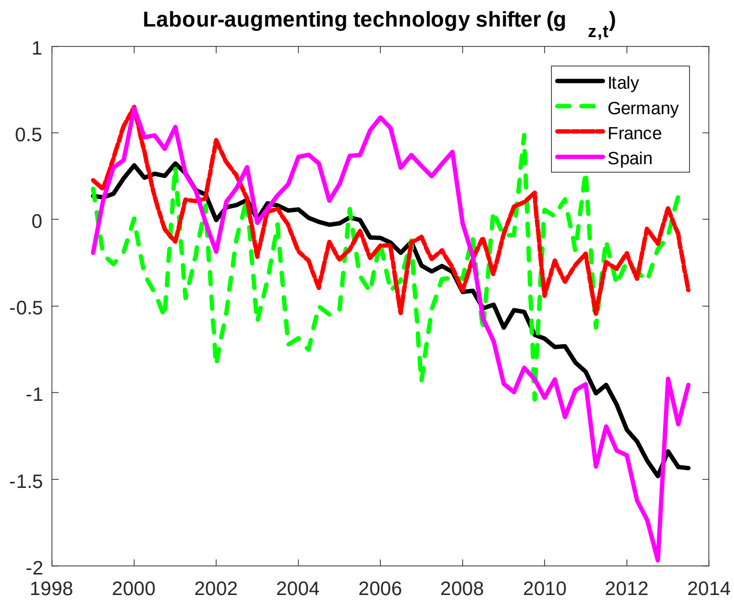

Figure 9: Estimated permanent technology shocks.

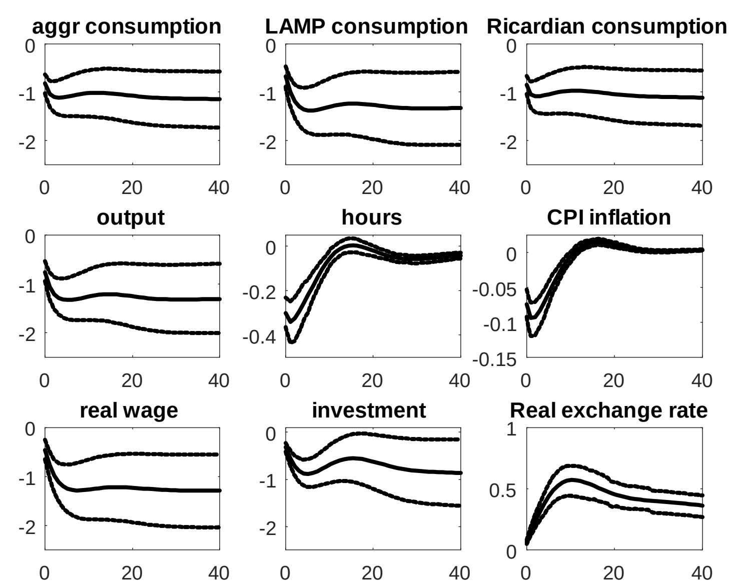

Figure 10: Impulse response functions to a negative permanent technology shock (Spain).

4 Conclusions

According to a popular wisdom, loose domestic credit conditions and undisciplined fiscal policies generated an illusory boom in the Eurozone southern economies, leading to competitiveness deterioration. These were the underlying factors that eventually led to sovereign bond crisis.

We cannot find support for this thesis in the cases of Spain and Italy, which account for 90% of the size of EMU southern economies. In fact, pre-2007 dynamics of growth and inflation in these two countries were not systematically stimulated by demand shocks. Further, the post-2010 severe output contraction experienced in these two countries was mainly determined by permanent adverse technology shocks. Thus the output losses experienced in these countries cannot be interpreted as a one-off price to pay in order to restore external competitiveness. Further, achieving cyclical recovery will not be sufficient to restore the relative income level that these countries had reached before the crisis.

To the extent that the slow down in productivity growth was the consequence of a credit crunch, our results suggest that macroeconomic policies should promote credit availability and favorable external financing conditions for innovative firms, and attempt to generate adequate domestic demand stimulus. In this regard it is interesting to note the relatively favorable growth performance of the Spanish economy in the last couple of years. In that country the government managed to free domestic banks from the burden of non-performing loans and was also allowed to escape the 3% deficit ceiling, whereas in Italy the solution to bank problems was delayed and the EMU rules limited fiscal flexibility due to the large stock of outstanding public debt.

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A Appendix

B The model

Each household supplies a continuum of size s of differentiated labor inputs, aggregated as follows

$$h_t^i = \left\{ \left(\frac{1}{s} \right)^{\frac{\lambda_t^w}{1+\lambda_t^w}} \int_0^s \left[h_t^i(j) \right]^{\frac{1}{1+\lambda_t^w}} dj \right\}^{1+\lambda_t^w} \tag{25}$$

Demand for labor type j is

$$h_t^j = \left(\frac{W_t^j}{W_t} \right)^{-\frac{1+\lambda_t^w}{\lambda_t^w}} h_t^d \tag{26}$$

where W_t^j is type j nominal wage and $W_t = \left[\frac{1}{s} \int_0^s \left(W_t^j \right)^{\frac{1}{\lambda_t^w}} dj \right]^{\lambda_t^w}$ is the aggregate nominal wage index. In each labor market j, wage-setting decisions are delegated to a monopolistic union and households supply the amount of labor that firms demand at W_t^j. Following Galì et al. (2007) we assume that the fractions of Ricardian and non-Ricardian households is uniformly distributed across worker types. Since wage-setting decisions are centralized, this implies that households supply an identical amount of labor services in each labor market j.

B.1 Production

Perfectly competitive firms produce the consumption and final investment bundles, C_t and Q_t^I, using tradable (C_t^T,$Q_t^{I,T}$) and nontradable (C_t^N,$Q_t^{I,N}$) intermediate goods. Monopolistically competitive firms produce the domestic (C_t^H, $Q_t^{I,H}$) and imported (C_t^F, $Q_t^{I,F}$) intermediate goods used as inputs for tradable goods.[14] Domestic and foreign tradable goods are indexed by $h \in [0, s]$ and $f \in [s, 1]$ respectively. Similarly, nontradable goods are indexed by $n \in [0, s]$ and $n^* \in [s, 1]$

[14] We assume that the law of one price holds for traded goods sold in the two economies.

respectively.

$$C_t^H = \left[\left(\frac{1}{s}\right)^{\frac{\lambda_t^{p,H}}{1+\lambda_t^{p,H}}} \int_0^s C_t^H(h)^{\frac{1}{1+\lambda_t^{p,H}}}\, dh\right]^{1+\lambda_t^{p,H}} \tag{27}$$

$$C_t^F = \left[\left(\frac{1}{1-s}\right)^{\frac{\lambda_t^{p,F}}{1+\lambda_t^{p,F}}} \int_s^1 C_t^F(f)^{\frac{1}{1+\lambda_t^{p,F}}}\, df\right]^{1+\lambda_t^{p,F}} \tag{28}$$

$$Q_t^{I,H} = \left[\left(\frac{1}{s}\right)^{\frac{\lambda_t^{p,H}}{1+\lambda_t^{p,H}}} \int_0^s Q_t^{I,H}(h)^{\frac{1}{1+\lambda_t^{p,H}}}\, dh\right]^{1+\lambda_t^{p,H}} \tag{29}$$

$$Q_t^{I,F} = \left[\left(\frac{1}{1-s}\right)^{\frac{\lambda_t^{p,F}}{1+\lambda_t^{p,F}}} \int_s^1 Q_t^{I,F}(f)^{\frac{1}{1+\lambda_t^{p,F}}}\, df\right]^{1+\lambda_t^{p,F}} \tag{30}$$

$$C_t^N = \left[\left(\frac{1}{s}\right)^{\frac{\lambda_t^{p,N}}{1+\lambda_t^{p,N}}} \int_0^s C_t^N(n)^{\frac{1}{1+\lambda_t^{p,N}}}\, dn\right]^{1+\lambda_t^{p,N}} \tag{31}$$

$$Q_t^{I,N} = \left[\left(\frac{1}{s}\right)^{\frac{\lambda_t^{p,N}}{1+\lambda_t^{p,N}}} \int_0^s Q_t^{I,N}(n)^{\frac{1}{1+\lambda_t^{p,N}}}\, dn\right]^{1+\lambda_t^{p,N}} \tag{32}$$

The composite price indexes are defined as:

$$P_t^N = \left[\frac{1}{s}\int_0^s P_t^N(n)^{\frac{1}{\lambda_t^{p,N}}}\, dn\right]^{\lambda_t^{p,N}} \tag{33}$$

$$P_t^H = \left[\frac{1}{s}\int_0^s P_t^H(h)^{\frac{1}{\lambda_t^{p,H}}}\, dh\right]^{\lambda_t^{p,H}} \tag{34}$$

$$P_t^F = \left[\frac{1}{1-s}\int_s^1 P_t^F(f)^{\frac{1}{\lambda_t^{p,F}}}\, df\right]^{\lambda_t^{p,F}} \tag{35}$$

where $\lambda_t^{p,H}$, $\lambda_t^{p,F}$, $\lambda_t^{p,N}$ denote time-varying net price markups.
The consumption bundle is:

$$C_t = \left[\gamma_c^{\frac{1}{e}}\left(C_t^T\right)^{\frac{e-1}{e}} + (1-\gamma_c)^{\frac{1}{e}}\left(C_t^N\right)^{\frac{e-1}{e}}\right]^{\frac{e}{e-1}} \tag{36}$$

where C_t^T is

$$C_t^T = \left[\varpi^{\frac{1}{\upsilon}}\left(C_t^H\right)^{\frac{\upsilon-1}{\upsilon}} + (1-\varpi)^{\frac{1}{\upsilon}}\left(C_t^F\right)^{\frac{\upsilon-1}{\upsilon}}\right]^{\frac{\upsilon}{\upsilon-1}};\ \upsilon > 1 \tag{37}$$

Consumer price indexes $P_{C,t}$ and P_t^T are:

$$P_{C,t} = \left[\gamma_c\left(P_t^T\right)^{1-e} + (1-\gamma_c)\left(P_t^N\right)^{1-e}\right]^{\frac{1}{1-e}} \tag{38}$$

$$P_t^T = \left[\varpi\left(P_t^H\right)^{1-\upsilon} + (1-\varpi)\left(P_t^F\right)^{1-\upsilon}\right]^{\frac{1}{1-\upsilon}} \tag{39}$$

Similarly, for final investment goods we posit:

$$Q_t^I = \left[\gamma_i^{\frac{1}{e}}\left(Q_t^{I,T}\right)^{\frac{e-1}{e}} + (1-\gamma_i)^{\frac{1}{e}}\left(Q_t^{I,N}\right)^{\frac{e-1}{e}}\right]^{\frac{e}{e-1}} \tag{40}$$

$$Q_t^{I,T} = \left[\varpi^{\frac{1}{\upsilon}}\left(Q_t^{I,H}\right)^{\frac{\upsilon-1}{\upsilon}} + (1-\varpi)^{\frac{1}{\upsilon}}\left(Q_t^{I,F}\right)^{\frac{\upsilon-1}{\upsilon}}\right]^{\frac{\upsilon}{\upsilon-1}} \tag{41}$$

$$P_{I,t} = \left[\gamma_i\left(P_t^T\right)^{1-e} + (1-\gamma_i)\left(P_t^N\right)^{1-e}\right]^{\frac{1}{1-e}} \tag{42}$$

Demand functions are:

$$C_t^N = (1-\gamma_c)\left(\frac{P_t^N}{P_{C,t}}\right)^{-e} C_t \tag{43}$$

$$C_t^H = \varpi\gamma_c\left(\frac{P_t^H}{P_t^T}\right)^{-\upsilon}\left(\frac{P_t^T}{P_{C,t}}\right)^{-e} C_t \tag{44}$$

$$C_t^F = (1-\varpi)\,\gamma_c\left(\frac{P_t^F}{P_t^T}\right)^{-\upsilon}\left(\frac{P_t^T}{P_{C,t}}\right)^{-e} C_t \tag{45}$$

$$Q_t^{I,N} = (1-\gamma_i)\left(\frac{P_t^N}{P_{I,t}}\right)^{-e} Q_t^I \tag{46}$$

$$Q_t^{I,H} = \varpi\gamma_i\left(\frac{P_t^H}{P_t^T}\right)^{-\upsilon}\left(\frac{P_t^T}{P_{I,t}}\right)^{-e} Q_t^I \tag{47}$$

$$Q_t^{I,F} = (1-\varpi)\,\gamma_i\left(\frac{P_t^F}{P_t^T}\right)^{-\upsilon}\left(\frac{P_t^T}{P_{I,t}}\right)^{-e} Q_t^I \tag{48}$$

Total demand for domestically produced intermediate goods is

$$Y_t^H = C_t^H + Q_t^{I,H} + \frac{1-s}{s}\left(C_t^{H^*} + Q_t^{I,H^*}\right) \tag{49}$$

$$Y_t^N = C_t^N + Q_t^{I,N} + G_t \tag{50}$$

where $\left(C_t^{H^*} + Q_t^{I,H^*}\right)$ defines foreign demand for home tradables and G_t is public consumption demand which is assumed to fall entirely on nontradables.[15]

B.1.1 Intermediate goods

The representative firm uses the following production technology:

$$Y_t^{int} = \varepsilon_t^{a,int}[u_t^{int}K_t^{int}]^{\alpha_{int}}[z_t h_t^{int}]^{1-\alpha_{int}} - z_t\Phi_{int} \tag{51}$$

where $int = h,\ f,\ n,\ n^*$, Φ_{int} defines fixed costs of production, u_t^{int} is the degree of capacity utilization, K_t^{int} is the capital stock, $\varepsilon_t^{a,int}$ is a temporary technology shock, such that

$$\log\left(\varepsilon_t^{a,int}\right) = (1-\rho_{int})\log\left(\varepsilon^{a,int}\right) + \rho_{int}\log\left(\varepsilon_{t-1}^{a,int}\right) + \eta_t^{int} \tag{52}$$

[15] We also make the standard assumption that G_t and C_t^N are identically distributed on individual non tradable goods.

and $z_t = z_{t-1} g_{z,t}$ is a labour-augmenting non-stationary technology shifter where

$$\log\left(g_{z,t}\right) = \left(1 - \rho_{g_z}\right) \log\left(g_z\right) + \rho_{g_z} \log\left(g_{z,t-1}\right) + \eta_t^{g_z} \tag{53}$$

allows to incorporate permanent technology shocks. The nominal marginal cost is:

$$MC_t^{int} = \alpha_{int}^{-\alpha_{int}} \left(1 - \alpha_{int}\right)^{-(1-\alpha_{int})} \left(\varepsilon_t^{a,int}\right)^{-1} z_t^{-(1-\alpha_{int})} \left(R_t^k\right)^{\alpha_{int}} \left[\left(1 + \tau^{wf}\right) W_t\right]^{1-\alpha_{int}} \tag{54}$$

where R_t^k is the nominal rental rate of capital and τ^{wf} is a payroll tax.

Price setting Firms optimally reset their price with probability $\left(1 - \xi_p^{int}\right)$. Non-optimizing firms adopt the standard indexation scheme:

$$P_t^{int} = \pi_{int,t-1}^{\chi_p^{int}} \bar{\pi}^{1-\chi_p^{int}} P_{t-1}^{int} \tag{55}$$

where $\bar{\pi}$ is the monetary union trend inflation rate and $\pi_{int,t} = \frac{P_t^{int}}{P_{t-1}^{int}}$ is the sectorial inflation rate. Note that $\bar{\bar{\pi}} = 100(\bar{\pi} - 1)$.

The first order condition for the optimizing firm is:

$$E_t \sum_{k=0}^{\infty} \left(\xi_p^{int}\right)^s \Xi_{t,t+k} Y_{t+k}^{int} \left[\frac{\tilde{P}_t^{int} \pi_{N,t,t+k-1}^{\chi_p^{int}} \bar{\pi}^{1-\chi_p^{int}}}{P_{C,t+k}} - \left(1 + \lambda_{t+k}^{p,int}\right) \frac{MC_{t+k}^{int}}{P_{C,t+k}} \right] = 0 \tag{56}$$

where Y_{t+k}^{int} defines total demand for goods produced in the sector, $\Xi_{t,t+s}$ is the stochastic discount factor to be defined below and

$$\pi_{int,t,t+k-1} = \begin{cases} 1 & \text{for } k = 0 \\ \pi_{int,t} \cdot \pi_{int,t+1} \cdot \ldots \cdot \pi_{int,t+k-1} & \text{for } k = 1, 2 \ldots. \end{cases} \tag{57}$$

The sectorial price index is:

$$P_t^{int} = \left[\left(1 - \xi_p^{int}\right) \left(\tilde{P}_t^{int}\right)^{\frac{1}{\lambda_t^{p,int}}} + \xi_p^{int} \left(\pi_{int,t-1}^{\chi_p^{int}} \bar{\pi}^{1-\chi_p^{int}} P_{t-1}^{int}\right)^{\frac{1}{\lambda_t^{p,int}}} \right]^{\lambda_t^{p,int}} \tag{58}$$

Note that price-setting decisions are affected by shocks to the elasticity of substitution across goods, that we characterize as net markup shocks, assumed to follow an AR(1) process with i.i.d. Normal error term:

$$\log\left(\lambda_t^{p,int}\right) = \left(1 - \rho_{p,int}\right) \log\left(\lambda^{p,int}\right) + \rho_{p,int} \log\left(\lambda_{t-1}^{p,int}\right) + \eta_t^{p,int} \tag{59}$$

B.2 Households

Households preferences are characterized by non separability between consumption and labor effort (Smets and Wouters, 2005, 2007):

$$U_t^i\left(c_t^i, h_t^i\right) = \frac{1}{1-\sigma} \left(\frac{c_t^i}{c_{t-1}^{\zeta}} \right)^{1-\sigma} \exp\left[\frac{\sigma - 1}{1 + \phi_l} \left(h_t^i\right)^{1+\phi_l} \right] \tag{60}$$

where consumption variables, $c_t^i = \frac{C_t^i}{z_t}$ and $c_t = \frac{C_t}{z_t}$, are normalized with the technology shifter z_t to ensure that the model has a balanced growth path. Parameter $0 < \zeta < 1$ measures the degree of external habit in consumption.[16]

[16] The habits-in-ratio specification limits the possibility that a non-negligible share of non-Ricardian households causes indeterminacy. In empirical DSGE model the existence of a relatively large indeterminacy region may bias posterior estimates. See Albonico et al. (2014 and 2015), for an extended dicussion.

B.2.1 Non-Ricardian households

Non-Ricardian households consume their current disposable income

$$\left(1+\tau^{c}\right) P_{C,t} C_{t}^{rt}=\left(1-\tau^{l}-\tau^{wh}\right) W_{t} h_{t}+T R_{t}^{rt} \tag{61}$$

where τ^c, τ^l, τ^{wh}, TR^{rt}, respectively denote consumption and labor income tax rates, social contributions levied on labor incomes, and lump-sum transfers.

B.2.2 Ricardian households

Ricardian households allocate their resources between consumption C_t^o, investment in physical capital I_t^o, in public bonds B_{t+1}^o and in a portfolio of state-contingent securities, A_t, that allow Ricardian households in the two regions to engage in mutual risk sharing. Their budget constraint is:

$$\begin{aligned}\left(1+\tau^{c}\right) P_{C,t} C_{t}^{o}+P_{I,t} I_{t}^{o}+A_{t}+B_{t+1}^{o} \quad = \quad & A_{t-1}+R_{t-1} B_{t}^{o}+\left(1-\tau^{l}-\tau^{wh}\right) W_{t}^{o} h_{t}^{o}+D_{t}^{o} \\ & +\left(1-\tau^{k}\right)\left[\frac{R_{t}^{k}}{\varepsilon_{t-1}^{b}} u_{t}^{o}-a\left(u_{t}^{o}\right) P_{I,t}\right] K_{t}^{o}+\tau^{k} \delta P_{I,t} K_{t}^{o}+T_{t}^{o}\end{aligned} \tag{62}$$

where T_t^o denotes lump-sum taxes levied on Ricardian households and ε_t^b is a risk premium shock that creates a wedge between the return on capital accruing to the households and the price of capital paid by firms.[17]

$$\log\left(\varepsilon_{t}^{b}\right)=\left(1-\rho_{b}\right) \log\left(\varepsilon^{b}\right)+\rho_{b} \log\left(\varepsilon_{t-1}^{b}\right)+\eta_{t}^{b} ; \eta_{t}^{b} \sim N\left(0, \sigma_{b}^{2}\right) \tag{63}$$

Physical capital accumulates as follows:

$$K_{t+1}^{o}=(1-\delta) K_{t}^{o}+\varepsilon_{t}^{i}\left[1-S\left(\frac{I_{t}^{o}}{I_{t-1}^{o}}\right)\right] I_{t}^{o} \tag{64}$$

where δ is the depreciation rate and ε_t^i denotes an investment-specific technology shock:

$$\log\left(\varepsilon_{t}^{i}\right)=\left(1-\rho_{i}\right) \log\left(\varepsilon^{i}\right)+\rho_{i} \log\left(\varepsilon_{t-1}^{i}\right)+\eta_{t}^{i} ; \eta_{t}^{i} \sim N\left(0, \sigma_{i}^{2}\right) \tag{65}$$

The term $S\left(\frac{I_t^o}{I_{t-1}^o}\right)$ represents investment adjustment costs. The standard adjustment costs function is:

$$S\left(\frac{I_{t}^{o}}{I_{t-1}^{o}}\right)=\frac{\gamma_{I}}{2}\left(\frac{I_{t}^{o}}{I_{t-1}^{o}}-g_{z}\right)^{2} \tag{66}$$

Capital utilization costs are defined as in Christiano et al. (2005):

$$a\left(u_{t}^{o}\right)=\gamma_{u1}\left(u_{t}^{o}-1\right)+\frac{\gamma_{u2}}{2}\left(u_{t}^{o}-1\right)^{2} \tag{67}$$

Ricardian households maximize

$$E_{t} \sum_{k=0}^{\infty} \beta^{t} \varepsilon_{t+k}^{c} U_{k}^{o}\left(c_{t+k}^{o}, h_{t+k}^{o}\right) \tag{68}$$

[17] A similar kind of shock is introduced in Ratto et al. (2008) and Amano and Shukayev (2012).

subject to (62), (64), (66) and (67). Term ε_t^c is a preference shock affecting the subjective discount factor and evolving according to:

$$\log\left(\varepsilon_t^c\right) = (1-\rho_c)\log\left(\varepsilon^c\right) + \rho_c \log\left(\varepsilon_{t-1}^c\right) + \eta_t^c; \eta_t^c \sim N\left(0, \sigma_c^2\right) \tag{69}$$

$\Lambda_t^o / P_{C,t}$ and $\Lambda_t^o Q_t^o$ respectively define the Lagrange multipliers associated with (62) and (64). The first order conditions are:

$$\varepsilon_t^c \left(c_t^o\right)^{-\sigma} c_{t-1}^{\zeta(\sigma-1)} \exp\left(\frac{\sigma-1}{1+\phi_l}\left(h_t^o\right)^{1+\phi_l}\right)\frac{1}{z_t} = \Lambda_t^o\left(1+\tau^c\right) \tag{70}$$

$$R_t = \pi_{C,t+1}\frac{\Lambda_t^o}{\beta\Lambda_{t+1}^o} \tag{71}$$

$$\begin{aligned} \frac{P_{I,t}}{P_{C,t}} &= Q_t^o \varepsilon_t^i \left\{1 - \gamma_I\left(\frac{I_t}{I_{t-1}} - g_z\right)\frac{I_t}{I_{t-1}} - \frac{\gamma_I}{2}\left(\frac{I_t}{I_{t-1}} - g_z\right)^2\right\} \\ &+ \Xi_{t,t+1} Q_{t+1}^o \varepsilon_{t+1}^i \gamma_I \left(\frac{I_{t+1}}{I_t} - g_z\right)\left(\frac{I_{t+1}}{I_t}\right)^2 \end{aligned} \tag{72}$$

$$\Xi_{t,t+1}\left\{\left(1-\tau^k\right)\left[\frac{R_{t+1}^k}{\varepsilon_t^b P_{C,t+1}} u_{t+1}^o - \frac{P_{I,t+1}}{P_{C,t+1}} a\left(u_{t+1}^o\right)\right] + \tau^k \frac{P_{I,t+1}}{P_{C,t+1}}\delta + Q_{t+1}^o\left(1-\delta\right)\right\} = Q_t^o \tag{73}$$

$$\frac{R_t^k}{\varepsilon_{t-1}^b P_{C,t}} = \frac{P_{I,t}}{P_{C,t}}\left[\gamma_{u1} + \gamma_{u2}\left(u_t^o - 1\right)\right] \tag{74}$$

Λ_t^o represents the shadow price of a unit of consumption good, thus equation (70) shows the marginal utility of consumption out of income. Q_t^o measures the shadow price of a unit of investment good and $\Xi_{t,t+1} = \beta\frac{\Lambda_{t+1}^o}{\Lambda_t^o}$ is the stochastic discount factor.

Equations (72) and (73) are the first order conditions for investment and capital respectively. Equation (74) equals the return from capital utilization to its cost. The latter equation implies that u_t^o is identical across Ricardian households, so that $u_t^o = u_t$. Further, the sectorial degree of capital utilization is uniform.

Equation (75) is the standard risk-sharing condition between Ricardian households in the two economies:

$$RER_t = \frac{\Lambda_t^{o,*}}{\Lambda_t^o} \tag{75}$$

where $RER_t \equiv \frac{P_{C,t}^*}{P_{C,t}}$ is the real effective exchange rate and initial conditions have been normalized to one.[18]

B.3 Labor market

Wages are staggered à la Calvo (1983). Union j receives permission to optimally reset the nominal wage with probability $(1-\xi_w)$. Non-optimizing unions adjust the wage according to the following scheme:

$$W_t^j = g_{z,t}\pi_{C,t-1}^{\chi_w}\bar{\pi}^{1-\chi_w}W_{t-1}^j \tag{76}$$

[18] Eq. (75) implies that Ricardian households, by trading state-contingent assets A_t, commit themselves to transfer schemes that allow to smooth consumption levels unless a variation occurs in their relative price RER_t.

where $\pi_t = \frac{P_{C,t}}{P_{C,t-1}}$ is the gross rate of consumer price inflation in the region.

We assume that unions maximize a weighted average of the two households types' utility functions:

$$\max_{\tilde{W}_t^j} E_t \sum_{k=0}^{\infty} (\xi_w \beta)^k \left\{ (1-\theta) U_t^o \left(c_{t+k}^o, h_{t+k}^o \right) + \theta U_t^{rt} \left(c_{t+k}^{rt}, h_{t+k}^{rt} \right) \right\} \tag{77}$$

subject to the budget constraints (61), (62) and to

$$h_{t+k}^i = \frac{1}{s} h_{t+k}^d \int_0^s \left(\frac{\tilde{W}_t^j g_{z,t,t+k} \pi_{C,t,t+k-1}^{\chi_w} \bar{\pi}^{1-\chi_w}}{W_{t+k}} \right)^{-\frac{1+\lambda_{t+k}^w}{\lambda_{t+k}^w}} dj \tag{78}$$

where $g_{z,t,t+k} = \prod_{k=1}^{k} g_{z,t+k}$,

$$\pi_{C,t,t+k-1} = \begin{cases} 1 & \text{for } \; k = 0 \\ \pi_{C,t} \pi_{C,t+1} \ldots \pi_{C,t+k-1} & \text{for } k = 1, 2.... \end{cases} \tag{79}$$

and time varying λ_t^w allows to incorporate wage markup shocks:

$$\log \left(\lambda_t^w \right) = (1-\rho_w) \log \left(\lambda^w \right) + \rho_w \log \left(\lambda_{t-1}^w \right) + \eta_t^w ; \eta_t^w \sim N \left(0, \sigma_w^2 \right) \tag{80}$$

The union first order condition is:

$$\begin{aligned} 0 \;\; = \;\; & E_t \sum_{s=0}^{\infty} (\xi_w \beta)^s \varepsilon_t^c c_{t+s-1}^{\zeta(\sigma-1)} \exp \left(\frac{\sigma - 1}{1 + \phi_l} \left(h_{t+s} \right)^{1+\phi_l} \right) h_{t+s}^j \cdot \\ & \cdot \left\{ \begin{array}{c} \tilde{W}_t^j \frac{\left(1-\tau^l-\tau^{wh}\right) g_{z,t,t+s} \pi_{C,t,t+s-1}^{\chi_w} \bar{\pi}^{1-\chi_w}}{(1+\tau^c) P_{C,t+s} z_{t+s}} \left(1 - \frac{1+\lambda_{t+s}^w}{\lambda_{t+s}^w} \right) \left[(1-\theta) \left(c_{t+s}^o \right)^{-\sigma} + \theta \left(c_{t+s}^{rt} \right)^{-\sigma} \right] \\ + \frac{1+\lambda_{t+s}^w}{\lambda_{t+s}^w} \left[(1-\theta) \left(c_{t+s}^o \right)^{-\sigma} MRS_{t+s}^o + \theta \left(c_{t+s}^{rt} \right)^{-\sigma} MRS_{t+s}^{rt} \right] \end{array} \right\} \end{aligned} \tag{81}$$

where

$$MRS_t^i = c_t^i h_t^{\phi_l} ; \; i = o, rt \tag{82}$$

The aggregate nominal wage index is

$$W_t = \left[\xi_w \left(g_{z,t} \pi_{C,t-1}^{\chi_w} \bar{\pi}^{1-\chi_w} W_{t-1} \right)^{\frac{1}{\lambda_t^w}} + (1-\xi_w) \left(\tilde{W}_t \right)^{\frac{1}{\lambda_t^w}} \right]^{\lambda_t^w} \tag{83}$$

B.4 Aggregation

$$C_t = \theta C_t^{rt} + (1-\theta) C_t^o \tag{84}$$

$$K_t = (1-\theta) K_t^o \tag{85}$$

$$I_t = (1-\theta) I_t^o \tag{86}$$

$$B_t = (1-\theta) B_t^o \tag{87}$$

$$D_t = (1-\theta) D_t^o \tag{88}$$

$$TR_t = \theta TR_t^{rt}$$

$$T_t = (1-\theta) TR_t^o$$

B.5 Market clearing

The following market clearing conditions obtain.

$$Y_t = \frac{P_t^H}{P_{C,t}} Y_t^H + \frac{P_t^N}{P_{C,t}} Y_t^N \tag{89}$$

$$u_t K_t = u_t^N K_t^N + u_t^H K_t^H \tag{90}$$

$$h_t^d = h_t^N + h_t^H \tag{91}$$

$$h_t = s_{W,t} h_t^d \tag{92}$$

$$s_{W,t} = \frac{1}{s} \int_0^s \left(\frac{W_t^j}{W_t} \right)^{-\frac{1+\lambda_t^w}{\lambda_t^w}} dj \tag{93}$$

$$Q_t^I = I_t + a\left(u_t\right) K_t \tag{94}$$

B.6 Government

The domestic government budget constraint is:

$$\frac{P_{N,t}}{P_{C,t}} G_t + R_{t-1} b_t + \frac{TR_t}{P_{C,t}} = \left\{ \begin{array}{c} \frac{b_{t+1}}{\pi_{C,t}} + \frac{T_t}{P_{C,t}} + \tau^c C_t + \left(\tau^l + \tau^{wh} + \tau^{wf}\right) \frac{W_t}{P_{C,t}} h_t + \\ +\tau^k \left[\frac{R_t^k}{P_{C,t}} u_t - \left(a\left(u_t\right) + \delta\right) \frac{P_{I,t}}{P_{C,t}} \right] K_t \end{array} \right\} \tag{95}$$

where b_t define real debt and public consumption is driven by

$$\log\left(\frac{g_t - g}{y}\right) = \rho_G \log\left(\frac{g_{t-1} - g}{y}\right) + \eta_t^G; \eta_t^G \sim N\left(0, \sigma_G^2\right) \tag{96}$$

Note that lower case letters stand for variables adjusted for growth, i.e. $g_t = G_t/z_t$, and g, y define steady state values.

B.7 ECB policy

As in Christoffel et al. (2008), the common monetary authority sets the nominal interest rate according to the following log-linear Taylor rule:

$$\begin{aligned} \hat{R}_t^{ECB} &= \phi_R \hat{R}_{t-1}^{ECB} + \left(1 - \phi_R\right)\left(\phi_\pi \hat{\pi}_{t-1}^{EA} + \phi_y \hat{y}_t^{EA}\right) \\ &\quad + \phi_{\Delta\pi}\left(\hat{\pi}_t^{EA} - \hat{\pi}_{t-1}^{EA}\right) + \phi_{\Delta y}\left(\hat{y}_t^{EA} - \hat{y}_{t-1}^{EA}\right) + \hat{\varepsilon}_t^r \end{aligned} \tag{97}$$

where '^' denotes log deviations from steady state. $\pi_t^{EA} = \pi_{C,t}^s \left(\pi_{C,t}^*\right)^{1-s}$ is the Euro Area gross inflation rate and $y_t^{EA} = s y_t + (1-s)\, y_t^*$ is the Euro Area aggregate output.

B.8 Non linear equations

The model is adjusted for growth, to obtain a balanced growth equilibrium. Thus, all growing variables are divided by the level of the technology shifter. Lower case letters stand for detrended variables, for example, $y_t = \frac{Y_t}{z_t}$. We define $\lambda_t^o = \Lambda_t^o z_t$ (see Christoffel, Coenen and Warne (2008)). c_t^o and c_t are already expressed as stationary variables. We also define $r_t^k = \frac{R_t^k}{P_{C,t}}$, $w_t = \frac{W_t}{z_t P_{C,t}}$, $tr_t^{rt} = \frac{TR_t^{rt}}{z_t P_{C,t}}$, $t_t^{rt} = \frac{T_t^{rt}}{z_t P_{C,t}}$. In this way it is also possible to compute the steady state of the model. For each country, lower letters price variables with a tilde "$\sim$" stand for the optimal price relative to aggregate price of the sector, for example: $\frac{\tilde{P}_t^n}{P_t^N} = \tilde{p}_t^n$.

Moreover, it is possible to express all the equilibrium equations as functions of relative prices. In particular, we will adopt the following definitions.

Terms of trade:

$$tt_t = \frac{P_t^F}{P_t^H} = \frac{P_t^{F,*}}{P_t^{H,*}}$$

where the second equality comes from the law of one price assumption.

"Internal" exchange rates:

$$x_t = \frac{P_t^N}{P_t^T}$$

$$x_t^* = \frac{P_t^{N^*}}{P_t^{T^*}}$$

All prices are expressed in terms of tt_t, x_t and x_t^*.

B.8.1 Relative prices

Relative investment prices Home country:

$$\frac{P_{I,t}}{P_{C,t}} = \left[\frac{\gamma_i + (1-\gamma_i)(x_t)^{1-e}}{\gamma_c + (1-\gamma_c)(x_t)^{1-e}}\right]^{\frac{1}{1-e}}$$

Foreign country:

$$\frac{P_{I,t}^*}{P_{C,t}^*} = \left[\frac{\gamma_i^* + (1-\gamma_i^*)(x_t^*)^{1-e}}{\gamma_c^* + (1-\gamma_c^*)(x_t^*)^{1-e}}\right]^{\frac{1}{1-e}}$$

Relative non tradable prices Home country:

$$\frac{P_t^N}{P_{C,t}} = \frac{x_t}{\left[\gamma_c + (1-\gamma_c)(x_t)^{1-e}\right]^{\frac{1}{1-e}}}$$

Foreign country:

$$\frac{P_t^{N,*}}{P_{C,t}^*} = \frac{x_t^*}{\left[\gamma_c^* + (1-\gamma_c^*)(x_t^*)^{1-e}\right]^{\frac{1}{1-e}}}$$

Relative tradables prices Home country:

$$\frac{P_t^H}{P_{C,t}} = \left\{ \left[\varpi + (1-\varpi) (tt_t)^{1-\upsilon} \right]^{\frac{1}{1-\upsilon}} \left[\gamma_c + (1-\gamma_c) (x_t)^{1-e} \right]^{\frac{1}{1-e}} \right\}^{-1}$$

$$\frac{P_t^H}{P_t^T} = \left[\varpi + (1-\varpi) (tt_t)^{1-\upsilon} \right]^{-\frac{1}{1-\upsilon}}$$

$$\frac{P_t^F}{P_t^T} = \left[\varpi \left(\frac{1}{tt_t} \right)^{1-\upsilon} + (1-\varpi) \right]^{-\frac{1}{1-\upsilon}}$$

$$\frac{P_t^T}{P_{C,t}} = \left[\gamma_c + (1-\gamma_c) (x_t)^{1-e} \right]^{-\frac{1}{1-e}}$$

Foreign country:

$$\frac{P_t^{F,*}}{P_{C,t}^*} = \left\{ \left[(1-\varpi^*) \left(\frac{1}{tt_t} \right)^{1-\upsilon} + \varpi^* \right]^{\frac{1}{1-\upsilon}} \left[\gamma_c^* + (1-\gamma_c^*) (x_t^*)^{1-e} \right]^{\frac{1}{1-e}} \right\}^{-1}$$

$$\frac{P_t^{H,*}}{P_t^{T,*}} = \left[(1-\varpi^*) + \varpi^* (tt_t)^{1-\upsilon} \right]^{-\frac{1}{1-\upsilon}}$$

$$\frac{P_t^{F,*}}{P_t^{T,*}} = \left[(1-\varpi^*) \left(\frac{1}{tt_t} \right)^{1-\upsilon} + \varpi^* \right]^{-\frac{1}{1-\upsilon}}$$

$$\frac{P_t^{T,*}}{P_{C,t}^*} = \left[\gamma_c^* + (1-\gamma_c^*) (x_t^*)^{1-e} \right]^{-\frac{1}{1-e}}$$

B.8.2 Households

Home country:

$$\varepsilon_t^c (c_t^o)^{-\sigma} c_{t-1}^{\zeta(\sigma-1)} \exp\left(\frac{(\sigma-1)}{1+\phi_l} (h_t)^{1+\phi_l} \right) = \lambda_t^o (1+\tau^c) \tag{98}$$

$$R_t = \pi_{C,t+1} g_{z,t+1} \frac{\lambda_t^o}{\beta \lambda_{t+1}^o} \tag{99}$$

$$\begin{aligned} \left[\frac{\gamma_i + (1-\gamma_i)(x_t)^{1-e}}{\gamma_c + (1-\gamma_c)(x_t)^{1-e}} \right]^{\frac{1}{1-e}} \quad &= \quad Q_t^o \varepsilon_t^i \left\{ 1 - \gamma_I \left(g_{z,t} \frac{i_t}{i_{t-1}} - g_z \right) g_{z,t} \frac{i_t}{i_{t-1}} - \frac{\gamma_I}{2} \left(g_{z,t} \frac{i_t}{i_{t-1}} - g_z \right)^2 \right\} \\ &+ \frac{1}{g_{z,t+1}} \frac{\lambda_{t+1}^o}{\lambda_t^o} Q_{t+1}^o \varepsilon_{t+1}^i \beta \gamma_I \left(g_{z,t+1} \frac{i_{t+1}}{i_t} - g_z \right) \left(g_{z,t+1} \frac{i_{t+1}}{i_t} \right)^2 \end{aligned} \tag{100}$$

$$\frac{1}{g_{z,t+1}} \frac{\lambda_{t+1}^o}{\lambda_t^o} \beta \left\{ \begin{array}{c} \left(1-\tau^k\right) \left[\frac{r_{t+1}^k}{\varepsilon_t^b} u_{t+1} - \left[\frac{\gamma_i + (1-\gamma_i)(x_{t+1})^{1-e}}{\gamma_c + (1-\gamma_c)(x_{t+1})^{1-e}} \right]^{\frac{1}{1-e}} a(u_{t+1}) \right] \\ + \tau^k \delta \left[\frac{\gamma_i + (1-\gamma_i)(x_{t+1})^{1-e}}{\gamma_c + (1-\gamma_c)(x_{t+1})^{1-e}} \right]^{\frac{1}{1-e}} + Q_{t+1}^o (1-\delta) \end{array} \right\} = Q_t^o \tag{101}$$

$$\frac{r_t^k}{\varepsilon_{t-1}^b} = \left[\frac{\gamma_i + (1-\gamma_i)(x_t)^{1-e}}{\gamma_c + (1-\gamma_c)(x_t)^{1-e}}\right]^{\frac{1}{1-e}} [\gamma_{u1} + \gamma_{u2}(u_t - 1)] \tag{102}$$

$$k_{t+1} = (1-\delta)\frac{k_t}{g_{z,t}} + \varepsilon_t^i \left[1 - \frac{\gamma_I}{2}\left(g_{z,t}\frac{i_t}{i_{t-1}} - g_z\right)^2\right] i_t \tag{103}$$

$$(1+\tau^c)\, c_t^{rt} = \left(1 - \tau^l - \tau^{wh}\right) w_t h_t + tr_t^{rt} - t_t^{rt} \tag{104}$$

$$c_t = \theta c_t^{rt} + (1-\theta)\, c_t^o \tag{105}$$

$$tr_t = \theta tr_t^{rt} + (1-\theta)\, tr_t^o \tag{106}$$

$$t_t = \theta t_t^{rt} + (1-\theta)\, t_t^o \tag{107}$$

$$MRS_t^o = c_t^o \varepsilon_t^l (h_t)^{\phi_l} \tag{108}$$

$$MRS_t^{rt} = c_t^{rt} \varepsilon_t^l (h_t)^{\phi_l} \tag{109}$$

Foreign country:

$$\varepsilon_t^{c,*} (c_t^{o,*})^{-\sigma} \left(c_{t-1}^*\right)^{\zeta^*(\sigma^*-1)} \exp\left(\frac{(\sigma^*-1)}{1+\phi_l^*}(h_t^*)^{1+\phi_l^*}\right) = \lambda_t^{o,*}(1+\tau^{c,*}) \tag{110}$$

$$\begin{aligned}\left[\frac{\gamma_i^* + (1-\gamma_i^*)(x_t^*)^{1-e}}{\gamma_c^* + (1-\gamma_c^*)(x_t^*)^{1-e}}\right]^{\frac{1}{1-e}} &= Q_t^{o,*}\varepsilon_t^{i,*}\left\{1 - \gamma_I^*\left(g_{z,t}\frac{i_t^*}{i_{t-1}^*} - g_z\right) g_{z,t}\frac{i_t^*}{i_{t-1}^*} - \frac{\gamma_I^*}{2}\left(g_{z,t}\frac{i_t^*}{i_{t-1}^*} - g_z\right)^2\right\} \\ &\quad + \frac{1}{g_{z,t+1}}\frac{\lambda_{t+1}^{o,*}}{\lambda_t^{o,*}} Q_{t+1}^{o,*}\varepsilon_{t+1}^{i,*}\beta\gamma_I^*\left(g_{z,t+1}\frac{i_{t+1}^*}{i_t^*} - g_z\right)\left(g_{z,t+1}\frac{i_{t+1}^*}{i_t^*}\right)^2\end{aligned} \tag{111}$$

$$\frac{1}{g_{z,t+1}}\frac{\lambda_{t+1}^{o,*}}{\lambda_t^{o,*}}\beta\left\{\begin{array}{c}\left(1-\tau^{k,*}\right)\left[r_{t+1}^{k,*}u_{t+1}^* - \left[\frac{\gamma_i^* + \left(1-\gamma_i^*\right)(x_t^*)^{1-e}}{\gamma_c^* + (1-\gamma_c^*)(x_t^*)^{1-e}}\right]^{\frac{1}{1-e}} a\left(u_{t+1}^*\right)\right] \\ +\tau^{k,*}\delta^*\left[\frac{\gamma_i^* + \left(1-\gamma_i^*\right)(x_t^*)^{1-e}}{\gamma_c^* + (1-\gamma_c^*)(x_t^*)^{1-e}}\right]^{\frac{1}{1-e}} + Q_{t+1}^{o,*}(1-\delta^*)\end{array}\right\} = Q_t^{o,*} \tag{112}$$

$$r_t^{k,*} = \left[\frac{\gamma_i^* + (1-\gamma_i^*)(x_t^*)^{1-e}}{\gamma_c^* + (1-\gamma_c^*)(x_t^*)^{1-e}}\right]^{\frac{1}{1-e}} [\gamma_{u1}^* + \gamma_{u2}^*(u_t^* - 1)] \tag{113}$$

$$k_{t+1}^* = (1-\delta^*)\frac{k_t^*}{g_{z,t}} + \varepsilon_t^{i,*}\left[1 - \frac{\gamma_I^*}{2}\left(g_{z,t}\frac{i_t^*}{i_{t-1}^*} - g_z\right)^2\right] i_t^* \tag{114}$$

$$(1+\tau^{c,*})\, c_t^{rt,*} = \left(1 - \tau^{l,*} - \tau^{wh,*}\right) w_t^* h_t^* + tr_t^{rt,*} - t_t^{rt,*} \tag{115}$$

$$c_t^* = \theta^* c_t^{rt,*} + (1-\theta^*)\, c_t^{o,*} \tag{116}$$

$$tr_t^* = \theta^* tr_t^{rt,*} + (1-\theta^*)\, tr_t^{o,*} \tag{117}$$

$$t_t^* = \theta^* t_t^{rt,*} + (1-\theta^*)\, t_t^{o,*} \tag{118}$$

$$MRS_t^{o,*} = c_t^{o,*} \varepsilon_t^{l,*} (h_t^*)^{\phi_l^*} \tag{119}$$

$$MRS_t^{rt,*} = c_t^{rt,*} \varepsilon_t^{l,*} (h_t^*)^{\phi_l^*} \tag{120}$$

B.8.3 Risk sharing condition

$$RER_t = \kappa \frac{\lambda_t^{o,*}}{\lambda_t^o} \tag{121}$$

B.8.4 Wages

$$\begin{aligned} 0 \;=\;& E_t \sum_{s=0}^{\infty} (\xi_w \beta)^s \varepsilon_t^c c_{t+s-1}^{\zeta(\sigma-1)} \exp\left(\frac{(\sigma-1)}{1+\phi_l} (h_{t+s})^{1+\phi_l}\right) \left(\frac{\pi_{C,t,t+s-1}^{\chi_w} \bar{\pi}_{t,t+s}^{1-\chi_w}}{w_{t+s}\pi_{C,t,t+s}}\right)^{-\frac{1+\lambda_{t+s}^w}{\lambda_{t+s}^w}} h_{t+s}^d \cdot \\ & \cdot \left\{ \begin{array}{c} \tilde{w}_t \frac{\left(1-\tau^l-\tau^{wh}\right)\pi_{C,t,t+s-1}^{\chi_w}\bar{\pi}_{t,t+s}^{1-\chi_w}}{(1+\tau^c)\pi_{C,t,t+s}} \left[(1-\theta)\left(c_{t+s}^o\right)^{-\sigma} + \theta\left(c_{t+s}^{rt}\right)^{-\sigma}\right] \\ -\left(1+\lambda_{t+s}^w\right)\left[(1-\theta)\left(c_{t+s}^o\right)^{-\sigma} MRS_{t+s}^o + \theta\left(c_{t+s}^{rt}\right)^{-\sigma} MRS_{t+s}^{rt}\right] \end{array} \right\} \end{aligned} \tag{122}$$

$$\begin{aligned} 0 \;=\;& E_t \sum_{s=0}^{\infty} (\xi_w^* \beta)^s \varepsilon_t^{c,*} \left(c_{t+s-1}^*\right)^{\zeta(\sigma-1)} \exp\left(\frac{(\sigma^*-1)}{1+\phi_l} \left(h_{t+s}^*\right)^{1+\phi_l}\right) h_{t+s}^{d,*} \left(\frac{\left(\pi_{C,t,t+s-1}^*\right)^{\chi_w^*} \bar{\pi}_{t,t+s}^{1-\chi_w^*}}{w_{t+s}^*\pi_{C,t,t+s}^*}\right)^{-\frac{1+\lambda_{t+s}^{w,*}}{\lambda_{t+s}^{w,*}}} \\ & \left\{ \begin{array}{l} \tilde{w}_t^* \frac{\left(1-\tau^l-\tau^{wh}\right)\left(\pi_{C,t,t+s-1}^*\right)^{\chi_w^*}\bar{\pi}_{t,t+s}^{1-\chi_w^*}}{(1+\tau^c)\pi_{C,t,t+s}^*} \left[(1-\theta^*)\left(c_{t+s}^{o,*}\right)^{-\sigma^*} + \theta^*\left(c_{t+s}^{rt,*}\right)^{-\sigma^*}\right] \\ -\left(1+\lambda_{t+s}^{w,*}\right)\left[(1-\theta^*)\left(c_{t+s}^{o,*}\right)^{-\sigma^*} MRS_{t+s}^{o,*} + \theta^*\left(c_{t+s}^{rt,*}\right)^{-\sigma^*} MRS_{t+s}^{rt,*}\right] \end{array} \right\} \end{aligned} \tag{123}$$

$$1 = \xi_w \left(\frac{\pi_{C,t-1}^{\chi_w}\bar{\pi}_t^{1-\chi_w}}{\pi_{C,t}} \frac{w_{t-1}}{w_t}\right)^{\frac{1}{\lambda_t^w}} + (1-\xi_w)\left(\frac{\tilde{w}_t}{w_t}\right)^{\frac{1}{\lambda_t^w}} \tag{124}$$

$$1 = \xi_w^* \left(\frac{\left(\pi_{C,t-1}^*\right)^{\chi_w^*}\bar{\pi}_t^{1-\chi_w^*}}{\pi_{C,t}^*} \frac{w_{t-1}^*}{w_t^*}\right)^{\frac{1}{\lambda_t^{w,*}}} + (1-\xi_w^*)\left(\frac{\tilde{w}_t^*}{w_t^*}\right)^{\frac{1}{\lambda_t^{w,*}}} \tag{125}$$

B.8.5 Production

Non-tradable goods Home country:

$$\frac{u_t k_t^N}{h_t^N g_{z,t}} = \frac{\alpha_N}{(1-\alpha_N)} \frac{\left(1+\tau_t^{wf}\right) w_t}{r_t^k} \tag{126}$$

$$mc_t^N = \alpha_N^{-\alpha_N} (1-\alpha_N)^{-(1-\alpha_N)} \left(\varepsilon_t^{a,N}\right)^{-1} \left(r_t^k\right)^{\alpha_N} \left[\left(1+\tau^{wf}\right) w_t\right]^{1-\alpha_N} \tag{127}$$

$$s_{P,t}^N y_t^N = \varepsilon_t^{a,N} \left(\frac{u_t^N k_t^N}{g_{z,t}}\right)^{\alpha_N} \left(h_t^N\right)^{1-\alpha_N} - \Phi_N \tag{128}$$

$$s_{P,t}^N = \frac{1}{s} \int_0^s \left(\frac{P_t^n}{P_t^N}\right)^{-\frac{1+\lambda_t^{p,N}}{\lambda_t^{p,N}}} dn \tag{129}$$

$$\begin{aligned} 0 \quad = \quad & E_t \sum_{s=0}^{\infty} \left(\beta \xi_p^N\right)^s \lambda_{t+s}^o y_{t+s}^N \left(\frac{\pi_{N,t,t+s-1}^{\chi_p^N} \bar{\pi}_{t,t+s}^{1-\chi_p^N}}{\pi_{N,t,t+s}}\right)^{-\frac{1+\lambda_{t+s}^{p,N}}{\lambda_{t+s}^{p,N}}} \cdot \\ & \cdot \left[\tilde{p}_t^n \frac{\pi_{N,t,t+s-1}^{\chi_p^N} \bar{\pi}_{t,t+s}^{1-\chi_p^N}}{\pi_{N,t,t+s}} \frac{x_{t+s}}{\left[\gamma_c + (1-\gamma_c)(x_{t+s})^{1-e}\right]^{\frac{1}{1-e}}} - \left(1+\lambda_{t+s}^{p,N}\right) mc_{t+s}^N\right] \end{aligned} \tag{130}$$

$$1 = \left(1-\xi_p^N\right) \left(\tilde{p}_t^n\right)^{\frac{1}{\lambda_t^{p,N}}} + \xi_p^N \left(\frac{\pi_{N,t-1}^{\chi_p^N} \bar{\pi}_t^{1-\chi_p^N}}{\pi_{N,t}}\right)^{\frac{1}{\lambda_t^{p,N}}} \tag{131}$$

Foreign country:

$$\frac{u_t k_t^{N,*}}{h_t^{N,*} g_{z,t}} = \frac{\alpha_N^*}{(1-\alpha_N^*)} \frac{\left(1+\tau^{wf,*}\right) w_t^*}{r_t^{k,*}} \tag{132}$$

$$mc_t^{N,*} = (\alpha_N^*)^{-\alpha_N^*} (1-\alpha_N^*)^{-\left(1-\alpha_N^*\right)} \left(\varepsilon_t^{a,N*}\right)^{-1} \left(r_t^{k,*}\right)^{\alpha_N^*} \left[\left(1+\tau^{wf,*}\right) w_t^*\right]^{1-\alpha_N^*} \tag{133}$$

$$s_{P,t}^{N,*} y_t^{N,*} = \varepsilon_t^{a,N*} \left(\frac{u_t^{N,*} k_t^{N,*}}{g_{z,t}}\right)^{\alpha_N^*} \left(h_t^{N,*}\right)^{1-\alpha_N^*} - \Phi_N^* \tag{134}$$

$$s_{P,t}^{N,*} = \frac{1}{1-s} \int_s^1 \left(\frac{P_t^{n,*}}{P_t^{N,*}}\right)^{-\frac{1+\lambda_t^{p,N*}}{\lambda_t^{p,N*}}} dn \tag{135}$$

$$\begin{aligned} 0 \quad = \quad & E_t \sum_{s=0}^{\infty} \left(\beta \xi_p^{N,*}\right)^s \lambda_{t,t+s}^{o,*} y_{t+s}^N \left(\frac{\left(\pi_{N,t,t+s-1}^*\right)^{\chi_p^{N,*}} \bar{\pi}_{t,t+s}^{1-\chi_p^{N,*}}}{\pi_{N,t,t+s}^*}\right)^{-\frac{1+\lambda_{t+s}^{p,N*}}{\lambda_{t+s}^{p,N*}}} \cdot \\ & \cdot \left[\tilde{p}_t^{n,*} \frac{\left(\pi_{N,t,t+s-1}^*\right)^{\chi_p^{N,*}} \bar{\pi}_{t,t+s}^{1-\chi_p^{N,*}}}{\pi_{N,t,t+s}^*} \frac{x_{t+s}^*}{\left[\gamma_c^* + (1-\gamma_c^*)(x_{t+s}^*)^{1-e}\right]^{\frac{1}{1-e}}} - \left(1+\lambda_{t+s}^{p,N*}\right) mc_{t+s}^{N,*}\right] \end{aligned} \tag{136}$$

$$1=\left(1-\xi_p^{N,*}\right)\left(\tilde{p}_t^{n,*}\right)^{\frac{1}{\lambda_t^{p,N*}}}+\xi_p^{N,*}\left(\frac{\left(\pi_{N,t-1}^{*}\right)^{\chi_p^{N,*}}\bar{\pi}_t^{1-\chi_p^{N,*}}}{\pi_{N,t}^{*}}\right)^{\frac{1}{\lambda_t^{p,N*}}} \tag{137}$$

Tradable goods Home country:

$$\frac{u_t k_t^H}{h_t^H g_{z,t}}=\frac{\alpha_H}{(1-\alpha_H)}\frac{\left(1+\tau^{wf}\right)w_t}{r_t^k} \tag{138}$$

$$mc_t^H=\alpha_H^{-\alpha_H}\left(1-\alpha_H\right)^{-(1-\alpha_H)}\left(\varepsilon_t^{a,H}\right)^{-1}\left(r_t^k\right)^{\alpha_H}\left[\left(1+\tau^{wf}\right)w_t\right]^{1-\alpha_H} \tag{139}$$

$$s_{P,t}^H y_t^H=\varepsilon_t^{a,H}\left(\frac{u_t^H k_t^H}{g_{z,t}}\right)^{\alpha_H}\left(h_t^H\right)^{1-\alpha_H}-\Phi_H \tag{140}$$

$$s_{P,t}^H=\frac{1}{s}\int_0^s\left(\frac{P_t^h}{P_t^H}\right)^{-\frac{1+\lambda_t^{p,H}}{\lambda_t^{p,H}}}dh \tag{141}$$

$$\begin{aligned}0 \quad = \quad & E_t\sum_{s=0}^{\infty}\left(\beta\xi_p^H\right)^s\lambda_{t,t+s}^o y_{t+s}^H\left(\frac{\pi_{H,t,t+s-1}^{\chi_p^H}\bar{\pi}_{t,t+s}^{1-\chi_p^H}}{\pi_{H,t,t+s}}\right)^{-\frac{1+\lambda_{t+s}^{p,H}}{\lambda_{t+s}^{p,H}}}\cdot \\ & \cdot\left[\begin{array}{c}\tilde{p}_t^h\frac{\pi_{H,t,t+s-1}^{\chi_p^H}\bar{\pi}_{t,t+s}^{1-\chi_p^H}}{\pi_{H,t,t+s}}\left\{\left[\varpi+(1-\varpi)\left(tt_{t+s}\right)^{1-\upsilon}\right]^{\frac{1}{1-\upsilon}}\left[\gamma_c+(1-\gamma_c)\left(x_{t+s}\right)^{1-e}\right]^{\frac{1}{1-e}}\right\}^{-1} \\ -\left(1+\lambda_{t+s}^{p,H}\right)mc_{t+s}^H\end{array}\right]\end{aligned} \tag{142}$$

$$1=\left(1-\xi_p^H\right)\left(\tilde{p}_t^h\right)^{\frac{1}{\lambda_t^{p,H}}}+\xi_p^H\left(\frac{\pi_{H,t-1}^{\chi_p^H}\bar{\pi}_t^{1-\chi_p^H}}{\pi_{H,t}}\right)^{\frac{1}{\lambda_t^{p,H}}} \tag{143}$$

Foreign country:

$$\frac{u_t k_t^F}{h_t^F g_{z,t}}=\frac{\alpha_F}{(1-\alpha_F)}\frac{\left(1+\tau^{wf,*}\right)w_t^*}{r_t^{k,*}} \tag{144}$$

$$mc_t^F=\alpha_F^{-\alpha_F}\left(1-\alpha_F\right)^{-(1-\alpha_F)}\left(\varepsilon_t^{a,F}\right)^{-1}\left(r_t^{k,*}\right)^{\alpha_F}\left[\left(1+\tau^{wf,*}\right)w_t^*\right]^{1-\alpha_F} \tag{145}$$

$$s_{P,t}^F y_t^F=\varepsilon_t^{a,F}\left(\frac{u_t^F k_t^F}{g_{z,t}}\right)^{\alpha_F}\left(h_t^F\right)^{1-\alpha_F}-\Phi_F \tag{146}$$

$$s_{P,t}^F=\frac{1}{1-s}\int_s^1\left(\frac{P_t^f}{P_t^F}\right)^{-\frac{1+\lambda_t^{p,F}}{\lambda_t^{p,F}}}df \tag{147}$$

$$0 = E_t \sum_{s=0}^{\infty} \left(\beta \xi_p^F\right)^s \lambda_{t,t+s}^{o,*} y_{t+s}^F \left(\frac{\pi_{F,t,t+s-1}^{\chi_p^F} \bar{\pi}_{t,t+s}^{1-\chi_p^F}}{\pi_{F,t,t+s}} \right)^{-\frac{1+\lambda_{t+s}^{p,F}}{\lambda_{t+s}^{p,F}}} \cdot \tag{148}$$

$$\cdot \left[\begin{array}{c} \tilde{p}_t^f \frac{\pi_{F,t,t+s-1}^{\chi_p^F} \bar{\pi}_{t,t+s}^{1-\chi_p^F}}{\pi_{F,t,t+s}} \left\{ \left[(1-\varpi^*) \left(\frac{1}{tt_{t+s}}\right)^{1-\upsilon} + \varpi^* \right]^{\frac{1}{1-\upsilon}} \left[\gamma_c^* + (1-\gamma_c^*) \left(x_{t+s}^*\right)^{1-e} \right]^{\frac{1}{1-e}} \right\}^{-1} \\ - \left(1+\lambda_{t+s}^{p,F}\right) mc_{t+s}^f \end{array} \right]$$

$$1 = \left(1-\xi_p^F\right) \left(\tilde{p}_t^f\right)^{\frac{1}{\lambda_t^{p,F}}} + \xi_p^F \left(\frac{\pi_{F,t-1}^{\chi_p^F} \bar{\pi}_t^{1-\chi_p^F}}{\pi_{F,t}} \right)^{\frac{1}{\lambda_t^{p,F}}} \tag{149}$$

B.8.6 Demand functions

Home country:

$$c_t^N = (1-\gamma_c)(x_t)^{-e} \left[\gamma_c + (1-\gamma_c)(x_t)^{1-e}\right]^{\frac{e}{1-e}} c_t \tag{150}$$

$$c_t^H = \varpi \gamma_c \left[\varpi + (1-\varpi)(tt_t)^{1-\upsilon}\right]^{\frac{\upsilon}{1-\upsilon}} \left[\gamma_c + (1-\gamma_c)(x_t)^{1-e}\right]^{\frac{e}{1-e}} c_t \tag{151}$$

$$c_t^F = (1-\varpi)\gamma_c \left[\varpi \left(\frac{1}{tt_t}\right)^{1-\upsilon} + (1-\varpi)\right]^{\frac{\upsilon}{1-\upsilon}} \left[\gamma_c + (1-\gamma_c)(x_t)^{1-e}\right]^{\frac{e}{1-e}} c_t \tag{152}$$

$$q_t^{I,N} = (1-\gamma_i)(x_t)^{-e} \left[\gamma_i + (1-\gamma_i)(x_t)^{1-e}\right]^{\frac{e}{1-e}} q_t^I \tag{153}$$

$$q_t^{I,H} = \varpi \gamma_i \left[\varpi + (1-\varpi)(tt_t)^{1-\upsilon}\right]^{\frac{\upsilon}{1-\upsilon}} \left[\gamma_i + (1-\gamma_i)(x_t)^{1-e}\right]^{\frac{e}{1-e}} q_t^I \tag{154}$$

$$q_t^{I,F} = (1-\varpi)\gamma_i \left[\varpi \left(\frac{1}{tt_t}\right)^{1-\upsilon} + (1-\varpi)\right]^{\frac{\upsilon}{1-\upsilon}} \left[\gamma_i + (1-\gamma_i)(x_t)^{1-e}\right]^{\frac{e}{1-e}} q_t^I \tag{155}$$

Foreign country:

$$c_t^{N*} = (1-\gamma_c^*)(x_t^*)^{-e} \left[\gamma_c^* + (1-\gamma_c^*)(x_t^*)^{1-e}\right]^{\frac{e}{1-e}} c_t^* \tag{156}$$

$$c_t^{H*} = (1-\varpi^*)\gamma_c^* \left[(1-\varpi^*) + \varpi^*(tt_t)^{1-\upsilon}\right]^{\frac{\upsilon}{1-\upsilon}} \left[\gamma_c^* + (1-\gamma_c^*)(x_t^*)^{1-e}\right]^{\frac{e}{1-e}} c_t^* \tag{157}$$

$$c_t^{F*} = \varpi^* \gamma_c^* \left[(1-\varpi^*) \left(\frac{1}{tt_t}\right)^{1-\upsilon} + \varpi^*\right]^{\frac{\upsilon}{1-\upsilon}} \left[\gamma_c^* + (1-\gamma_c^*)(x_t^*)^{1-e}\right]^{\frac{e}{1-e}} c_t^* \tag{158}$$

$$q_t^{I,N*} = (1-\gamma_i^*)(x_t^*)^{-e} \left[\gamma_i^* + (1-\gamma_i^*)(x_t^*)^{1-e}\right]^{\frac{e}{1-e}} q_t^{I,*} \tag{159}$$

$$q_t^{I,H*} = (1-\varpi^*)\gamma_i^* \left[(1-\varpi^*) + \varpi^*(tt_t)^{1-\upsilon}\right]^{\frac{\upsilon}{1-\upsilon}} \left[\gamma_i^* + (1-\gamma_i^*)(x_t^*)^{1-e}\right]^{\frac{e}{1-e}} q_t^{I,*} \tag{160}$$

$$q_t^{I,F*} = \varpi^* \gamma_i^* \left[(1-\varpi^*) \left(\frac{1}{tt_t} \right)^{1-\upsilon} + \varpi^* \right]^{\frac{\upsilon}{1-\upsilon}} \left[\gamma_i^* + (1-\gamma_i^*) (x_t^*)^{1-e} \right]^{\frac{e}{1-e}} q_t^{I,*} \tag{161}$$

B.8.7 Relative price of non tradable goods

$$\frac{x_t}{x_{t-1}} = \frac{\pi_t^N}{\pi_t^T} \tag{162}$$

$$\frac{x_t^*}{x_{t-1}^*} = \frac{\pi_t^{N,*}}{\pi_t^{T,*}} \tag{163}$$

B.8.8 Tradables inflation

Home country:

$$\pi_t^T = \left[\varpi + (1-\varpi) (tt_t)^{1-\upsilon} \right]^{\frac{1}{1-\upsilon}} \left[\varpi + (1-\varpi) (tt_{t-1})^{1-\upsilon} \right]^{-\frac{1}{1-\upsilon}} \pi_t^H \tag{164}$$

Foreign country:

$$\pi_t^T = \left[(1-\varpi^*) \left(\frac{1}{tt_t} \right)^{1-\upsilon} + \varpi^* \right]^{\frac{1}{1-\upsilon}} \left[(1-\varpi^*) \left(\frac{1}{tt_{t-1}} \right)^{1-\upsilon} + \varpi^* \right]^{-\frac{1}{1-\upsilon}} \pi_t^F \tag{165}$$

B.8.9 Market clearing

Home country:

$$y_t^N = c_t^N + q_t^{I,N} + g_t \tag{166}$$

$$y_t^H = c_t^H + \frac{1-s}{s} c_t^{H*} + q_t^{I,H} + \frac{1-s}{s} q_t^{I,H*} \tag{167}$$

$$y_t = \frac{\left[\varpi + (1-\varpi) (tt_t)^{1-\upsilon} \right]^{-\frac{1}{1-\upsilon}} y_t^H + x_t y_t^N}{\left[\gamma_c + (1-\gamma_c) (x_t)^{1-e} \right]^{\frac{1}{1-e}}} \tag{168}$$

$$k_t = k_t^N + k_t^H \tag{169}$$

$$h_t^d = h_t^N + h_t^H \tag{170}$$

$$h_t = s_{W,t} h_t^d \tag{171}$$

$$s_{W,t} = \frac{1}{s} \int_0^s \left(\frac{W_t^j}{W_t} \right)^{-\frac{1+\lambda_t^w}{\lambda_t^w}} dj \tag{172}$$

$$q_t^I = i_t + a(u_t) \frac{k_t}{g_{z,t}} \tag{173}$$

Foreign country:

$$y_t^{N*} = c_t^{N*} + q_t^{I,N,*} + g_t^* \tag{174}$$

$$y_t^F = \frac{s}{1-s} c_t^F + c_t^{F^*} + \frac{s}{1-s} q_t^{I,F} + q_t^{I,F^*} \tag{175}$$

$$y_t^* = \frac{\left[(1-\varpi^*) \left(\frac{1}{tt_t} \right)^{1-\upsilon} + \varpi^* \right]^{-\frac{1}{1-\upsilon}} y_t^F + x_t^* y_t^{N*}}{\left[\gamma_c^* + (1-\gamma_c^*) (x_t^*)^{1-e} \right]^{\frac{1}{1-e}}} \tag{176}$$

$$k_t^* = k_t^{N,*} + k_t^F \tag{177}$$

$$h_t^{d,*} = h_t^{N,*} + h_t^F \tag{178}$$

$$h_t^* = s_{W,t}^* h_t^{d,*} \tag{179}$$

$$s_{W,t}^* = \frac{1}{1-s} \int_s^1 \left(\frac{W_t^{j,*}}{W_t^*} \right)^{-\frac{1+\lambda_t^{w*}}{\lambda_t^{w*}}} dj \tag{180}$$

$$q_t^{I,*} = i_t^* + a(u_t^*) \frac{k_t^*}{g_{z,t}} \tag{181}$$

B.8.10 CPI Inflation

Home country:

$$\pi_{C,t} = \left[\frac{\gamma_c (\pi_t^T)^{1-e} + (1-\gamma_c) (\pi_t^N)^{1-e} (x_{t-1})^{1-e}}{\gamma_c + (1-\gamma_c)(x_{t-1})^{1-e}} \right]^{\frac{1}{1-e}} \tag{182}$$

Foreign country:

$$\pi_{C,t}^* = \left[\frac{\gamma_c^* \left(\pi_t^{T,*} \right)^{1-e} + (1-\gamma_c^*) \left(\pi_t^{N,*} \right)^{1-e} \left(x_{t-1}^* \right)^{1-e}}{\gamma_c^* + (1-\gamma_c^*) \left(x_{t-1}^* \right)^{1-e}} \right]^{\frac{1}{1-e}} \tag{183}$$

B.8.11 Real exchange rate

$$RER_t = \frac{\left[\gamma_c^* + (1-\gamma_c^*) \left(x_t^* \right)^{1-e} \right]^{\frac{1}{1-e}}}{\left[\gamma_c + (1-\gamma_c)(x_t)^{1-e} \right]^{\frac{1}{1-e}}} \frac{\left[(1-\varpi^*) + \varpi^* (tt_t)^{1-\upsilon} \right]^{\frac{1}{1-\upsilon}}}{\left[\varpi + (1-\varpi)(tt_t)^{1-\upsilon} \right]^{\frac{1}{1-\upsilon}}} \tag{184}$$

$$\frac{RER_t}{RER_{t-1}} = \frac{\frac{P_{C,t}^*}{P_{C,t}}}{\frac{P_{C,t-1}^*}{P_{C,t-1}}} = \frac{\pi_{C,t}^*}{\pi_{C,t}} \tag{185}$$

B.8.12 Resource constraints

$$
\begin{aligned}
y_t &= c_t + \frac{P_{I,t}}{P_{C,t}} q_t^I + \frac{P_t^N}{P_{C,t}} g_t + \frac{P_t^H}{P_{C,t}} \frac{1-s}{s} \left(c_t^{H,*} + q_t^{I,H,*} \right) - \frac{P_t^F}{P_{C,t}} \left(c_t^F + q_t^{I,F} \right) \\
&= c_t + \frac{P_{I,t}}{P_{C,t}} q_t^I + \frac{P_t^N}{P_{C,t}} g_t + \frac{P_t^H}{P_{C,t}} \frac{1-s}{s} ex_t - \frac{P_t^F}{P_{C,t}} im_t
\end{aligned}
$$

where ex_t stands for exports and im_t for imports.

Similarly, for the foreign country we obtain:

$$
\begin{aligned}
y_t^* &= c_t^* + \frac{P_{I,t}^*}{P_{C,t}^*} q_t^{I,*} + \frac{P_t^{N,*}}{P_{C,t}^*} g_t^* + \frac{P_t^F}{P_{C,t}^*} \frac{s}{1-s} \left(c_t^F + q_t^{I,F} \right) - \frac{P_t^H}{P_{C,t}^*} \left(c_t^{H,*} + q_t^{I,H,*} \right) \\
&= c_t^* + \frac{P_{I,t}^*}{P_{C,t}^*} q_t^{I,*} + \frac{P_t^{N,*}}{P_{C,t}^*} g_t^* + \frac{P_t^F}{P_{C,t}^*} \frac{s}{1-s} ex_t^* - \frac{P_t^H}{P_{C,t}^*} im_t^* \\
&= c_t^* + \frac{P_{I,t}^*}{P_{C,t}^*} q_t^{I,*} + \frac{P_t^{N,*}}{P_{C,t}^*} g_t^* + \frac{P_t^F}{P_{C,t}^*} \frac{s}{1-s} im_t - \frac{P_t^H}{P_{C,t}^*} ex_t
\end{aligned}
$$

where ex_t^* stands for the foreign country exports and im_t^* for the foreign country imports and $ex_t^* = im_t$, $im_t^* = ex_t$.

Definition of exports and imports

$$
ex_t = c_t^{H,*} + q_t^{I,H,*}
$$

$$
im_t = c_t^F + q_t^{I,F}
$$

B.9 Steady state

We assume that exogenous shocks are equal to one in steady state. For utilization, $u = 1$ so that $a(u) = 0$.

We make some simplifying assumptions, which enable us to find the steady state analytically.

In each country, we impose the same price markup ($\lambda^{p,H} = \lambda^{p,N} = \lambda_p$, $\lambda^{p,F} = \lambda^{p,N,*} = \lambda_p^*$) and the same shares of capital in production ($\alpha_H = \alpha_N$, $\alpha_F = \alpha_{N,*}$) in both sectors. This implies that $P^H = P^N$ and $P^F = P^{N,*}$ in steady state. We set the fixed costs so that steady state profits are zero, which implies also that $\frac{y^N + \Phi_N}{y^N} = \frac{y^H + \Phi_H}{y^H} = 1 + \lambda_p$.

Moreover, we impose that in steady state quantities of exports and imports correspond in steady state, so that home tradable prices are equal to foreign tradable prices ($P^H = P^F$) and finally steady state net exports are equal to zero. Thus:

$$
\frac{\frac{1-s}{s} \left(c^{H,*} + q^{I,H,*} \right)}{\left(c^F + q^{I,F} \right)} = \frac{P^F}{P^H} = 1
$$

This in turn implies that $P^F = P^H = P^N = P^{N,*} = P^T = P^{T,*} = P_C = P_C^* = P_I = P_I^*$, thus all relative prices are equal to 1 in steady state. Thus, also tt_t, x_t, x_t^* are 1 in steady state.