Home biased expectations and macroeconomic imbalances in a monetary union*

Dennis Bonam†[1] and Gavin Goy‡[2]

[1]De Nederlandsche Bank

[2]University of Amsterdam

August 30, 2017

Abstract

Under monetary union, economic dynamics may diverge across countries due to regional inflation differentials and a pro-cyclical real interest rate channel, yet stability is generally ensured through endogenous adjustment of the real exchange rate. The speed of adjustment depends, inter alia, on the way agents form expectations. We propose a model in which agents' expectations are largely based on domestic variables, and less so on foreign variables. We show that such home bias in expectations strengthens the real interest rate channel and causes country-specific shocks to generate larger and more prolonged macroeconomic imbalances.

JEL Classification: E03, F44, F45

Keywords: monetary union, macroeconomic imbalances, home biased expectations, E-stability

*We would like to thank Peter van Els, Jakob de Haan, Gabriele Galati, Jeroen Hessel, Bart Hobijn, Cars Hommes, Kostas Mavromatis and Manu de Veirman for helpful comments. Also, we are thankful to participants of the QED Jamboree 2017 in Paris, the Tinbergen PhD Jamboree in Amsterdam and the CEF Conference 2017 in New York, as well as seminar participants at De Nederlandsche Bank and the University of Amsterdam for fruitful discussions. All errors are our own. The views expressed do not necessarily reflect the official position of De Nederlandsche Bank or the Eurosystem.

†Contact: Dennis Bonam (d.a.r.bonam@dnb.nl), De Nederlandsche Bank, Econometrics and Modeling dept., Westeinde 1, 1017 ZN, Amsterdam, The Netherlands.

‡Contact: Gavin Goy (g.w.goy@uva.nl), University of Amsterdam, Roetersstraat 11, 1018 WD Amsterdam, The Netherlands.

1 Introduction

Expectations play an important role in the design and evaluation of economic policy, which explains their prominence in many macroeconomic models. Inflation expectations, for instance, typically feed into actual inflation through price-setting and wage bargaining, and are therefore closely monitored by central banks who aim to maintain price stability (Orphanides and Williams, 2005; Bernanke, 2007; Mishkin, 2007). Recent studies suggest that inflation expectations in the euro area are well anchored around the European Central Bank's inflation target of close to, yet below, two percent (e.g. Beechey et al., 2011, Ehrmann et al., 2011, Galati et al., 2011, ECB, 2012, Autrup and Grothe, 2014). However, these results are often based on aggregate expectations and do not take into account potentially important disparities across member countries.

The top panel of Figure 1 shows the evolution of one-year ahead inflation expectations of households from different euro area countries, extracted from the European Commission's Consumer Survey. As the figure makes clear, inflation expectations differ strongly across countries.[1] Moreover, the dispersion in expectations is most pronounced during the first years of European monetary unification, a period characterized by large macroeconomic imbalances within the union. The bottom panel plots the regional dispersion in inflation expectations against the dispersion in countries' external balance as a percentage of GDP, both measured by the unweighted cross-country standard deviation, for the period 1997Q1 to 2007Q4. The figure suggests that, during this period, larger cross-country differences in inflation expectations have been accompanied by more dispersed external balance ratios.

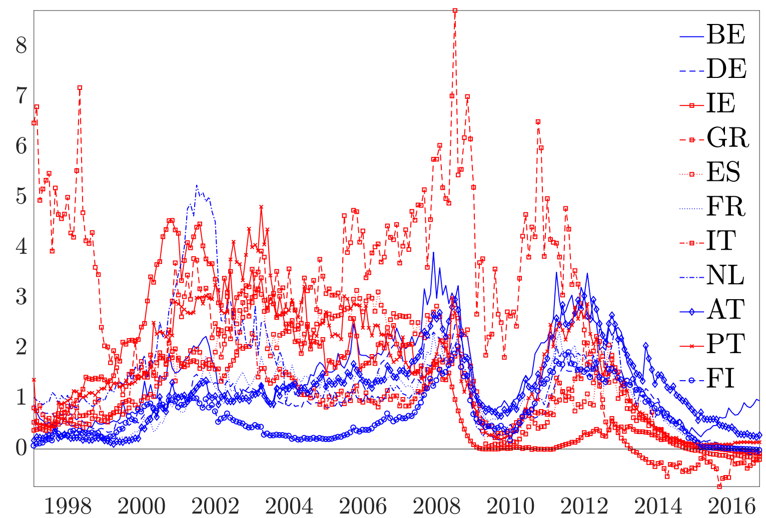
In this paper, we aim to explain this empirically observed relationship between cross-country heterogeneity in expectations and macroeconomic imbalances in a monetary union.[2]

[1]Importantly, when asking about expected changes in the price level, the survey is not specific about the *type* of price level, be it domestic prices, non-domestic prices, or some consumption weighted price index, yet rather refers to a general concept of the price level.

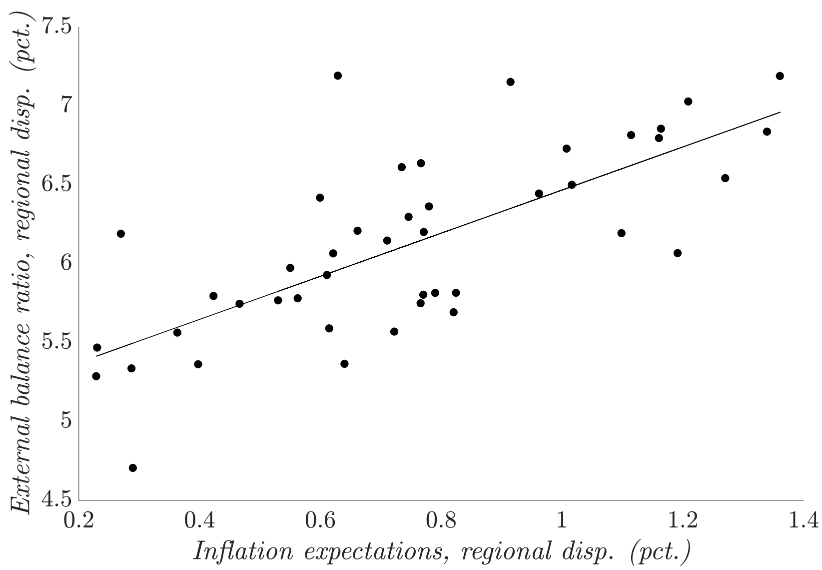
[2]Throughout the paper, we refer to macroeconomic imbalances as the dispersion in the external (or trade) balance across countries. One could, however, also think of macroeconomic imbalances as the dispersion in current account balances across countries.

Figure 1: Expectations and cross-country dispersion in the euro area

(*a*) Inflation expectations (one-year ahead) based on survey responses (pct.)



(*b*) Regional dispersion in inflation expectations and external balance ratio, 1997Q1-2007Q4



Notes. Panel (*a*): The qualitative survey responses were extracted from the European Commission's Consumer Survey and quantified using the probability approach as described by Dias et al. (2010). See the Online Appendix for more details. Panel (*b*): In the calculation of the standard deviation in inflation expectations and the external balance ratio, we excluded Greece due to some episodes of very high Greek inflation expectations. *Source*: European Commission's Consumer Survey (inflation expectations), OECD Quarterly National Accounts dataset (external balance) and own calculations.

We propose a New Keynesian model for a two-country monetary union in which expectations are backward looking and updated periodically using a recursive least squares algorithm, as in Evans and Honkapohja (2001). Moreover, we assume that agents, while acquiring information, focus more strongly on domestic variables than on foreign variables. In modeling this *home bias* in expectations, we remain agnostic on its many potential sources and rather choose a parsimonious approach in which the home bias is exogenous. It is because of this home bias that expectations may differ across member states in our model. We show that a stronger degree of home bias in expectations leads to more amplified country-specific disturbances and thereby larger and more prolonged macroeconomic imbalances.

To make this result clear, we use our model to study the effects of a country-specific demand shock. If a country experiences a positive demand shock, inflation rises which raises inflation expectations (assuming some inertia or backward-looking behavior) and lowers the ex-ante real interest rate, which in turn fuels private spending and puts further pressure on inflation (expectations), which further lowers the real interest rate. We refer to this pro-cyclical channel as the *real interest rate channel*. Meanwhile, the rise in inflation leads to an appreciation of the real exchange rate, which reduces net exports and domestic output. Therefore, the real interest rate channel is countered by a counter-cyclical *real exchange rate channel*. The opposite occurs if a country were to experience a negative demand shock. In general, such country-specific shocks are amplified under monetary union, since the common central bank does not directly respond to national inflation, yet instead combats union-wide inflation.

The strength of the real interest rate channel relative to the real exchange rate channel depends, inter alia, on how expectations are formed and the extent by which agents internalize the countervailing effects of the real exchange rate. The backward-looking nature of expectations assumed in our model reinforces the feedback between inflation and inflation expectations, and thereby strengthens the real interest rate channel. At the same time, when expectations are home biased, agents do not fully observe the relative change in domestic versus foreign prices and thus underestimate the impact of the real exchange rate on the domestic economy. The home bias in expectations therefore weakens the real exchange rate channel and slows down the process of macroeconomic realignment. As a consequence, country-specific disturbances generate larger and more prolonged macroeconomic imbalances if expectations are home biased than if expectations are rational or free from home bias.

Our findings offer a possible explanation for the observed positive relationship between the dispersion in inflation expectations and external imbalances in the euro area, as shown in Figure 1: in times of large country-specific disturbances, expectations may have become more home biased, causing economic divergences within the euro area to be more pronounced due to the mechanism described above. The results therefore warrant greater attention from policymakers to the development of national inflation expectations, even if aggregate expectations seem relatively stable. In fact, the degree of home bias in expectations across member countries helps determine whether inflation differentials are only temporary, and part of an endogenous adjustment process, or structural and potentially unstable. We provide empirical estimates for the degree of home bias in expectations for a group of euro area countries and find that home bias has indeed been strongest in times when external imbalances were particularly large.

Examination of the model's stability conditions under home biased expectations reveals important implications for monetary policy. We find that a higher degree of home bias in expectations reduces the likelihood that expectations converge to a fixed point. Because, in our model, the bias in expectations is permanent, beliefs do not saddle down to the rational expectations equilibrium and so we define stability in expectations as convergence towards a so-called Restricted Perception Equilibrium or RPE (see also Evans and Honkapohja, 2001, Chapters 3.6 and 13). Numerical simulations show that the RPE may not be reached if expectations are home biased, even in parameter regions where monetary policy yields a determinate rational expectations equilibrium. Intuitively, since the home bias in expectations increases the relative strength of the real interest rate channel, inflation expectations are more likely to become unanchored. Therefore, to guarantee an equilibrium that is both determinate and stable under learning, the central bank must adopt a more aggressive monetary stance with regards to inflation, beyond what is suggested by the Taylor-principle.

Our paper closely relates to the debate on the Walters critique (Walters, 1994) which centers around the power struggle between the real interest rate and real exchange rate channels, and the stability of monetary unions. The issue of internal adjustment under monetary union has been studied, among others, by Angeloni and Ehrmann (2007), Deroose et al. (2008) and Allsopp and Vines (2010). In accordance with our results, Angeloni and Ehrmann (2007) show that inflation differentials in the euro area are amplified by the pro-cyclical real interest rate channel and find a dampening effect through changes in price competitiveness within the monetary union. Interestingly, the authors also mention the possibility of "*a strong "home bias" in the mechanism driving inflationary expectations in the national economies*" that could amplify inflation differentials (Angeloni and Ehrmann, 2007, p. 8). Deroose et al. (2008) and Allsopp and Vines (2010) focus on national (fiscal) policies and reforms that help increase market flexibility and thereby strengthen the real exchange rate channel.

Although there is ample empirical support for the presence of heterogeneity in expectations (e.g. Carroll, 2003, Mankiw and Reis, 2002, Pfajfar and Santoro, 2010, and Madeira and Zafar, 2015), only a few studies relate such heterogeneity to the Walters critique. A notable exception is Carlin (2013), who uses a theoretical model to show that countries populated by non-rational wage-setters are more vulnerable to pro-cyclical real interest rate effects than countries in which wage-setting is more consistent with rational expectations. Likewise, Toroj (2010) uses a New Keynesian model to study asymmetric disturbances in a monetary union and finds macroeconomic volatility to be higher when agents use different (simple) forecasting rules than when expectations are rational. A key difference between these studies and the present paper, is that we allow agents' learning methods to be *symmetric* across countries, since it is not a priori clear why these should be asymmetric. If expectations were to differ across countries, we believe a more natural and straightforward explanation would be that agents use different information sets, simply because agents are more likely to notice local rather than foreign shocks.

Our paper also connects to the literature on heterogeneous expectations in New Keynesian models and their implications for policy analysis (e.g. Bullard and Mitra, 2002; Evans and Honkapohja, 2003; Evans and McGough, 2007). Many papers that fall under this branch of the literature study heterogeneity in expectations arising from differences in the forecasting heuristics used by agents, and typically feature closed economy models. For instance, in Branch and McGough (2009), Massaro (2013) and Gasteiger (2014), only a fraction of the population operate under rational expectations, while others are boundedly rational. A general finding from these studies is that such heterogeneity in expectations can result in economic instability under standard monetary policy rules (see also Berardi, 2009). In Honkapohja and Mitra (2006), among others, agents are allowed to use different forecasting methods that yield dispersed predictions about future economic outcomes. In this case, one is able to capture heterogeneity in expectations not only across individuals, but also across time. Relatedly, Mankiw and Reis (2002) present a model of sticky information in which expectations are updated infrequently by a random, yet constant, fraction of the population. The authors show that such a model can resolve some business cycle puzzles, such as the persistence of inflation, the delayed and gradual effect of monetary policy on inflation and the contractionary effects of (surprise) disinflations.

In this paper, we consider information asymmetry as a possible source of heterogeneity in expectations. For our purposes, all that is required is that expectations are formed in a way that ignores (at least to some extent) information on foreign variables. This is ensured through a weighting down of foreign variables in agents' information sets. We calibrate this weight based on an estimated adaptive learning model for the euro area. Armed with this novel measure for home bias in expectations, we study its role for macroeconomic imbalances in a monetary union, which is the primary contribution of this paper. In addition, from a more methodological point of view, our paper formulates a novel RPE and analyzes its stability under learning.[3]

The rest of the paper is structured as follows. In the next section, we elaborate on the intuition underlying our assumption of home bias in expectations. We also present a case study in which we show that inflation expectations became more home biased in Spain during the Spanish housing boom that preceded the Great Recession. In Section 3, we provide a brief overview of the model, explain the adaptive learning methodology of Evans and Honkapohja (2001), and discuss how home bias in expectations is introduced to the model. The results, based on impulse responses, are discussed in Section 4 and the stability analysis is performed in Section 5. Finally, Section 6 concludes.

2 Home biased expectations and the Spanish housing boom

Expectations regarding future economic conditions may differ across agents and countries for various reasons. As discussed in the introduction, several studies have attempted to capture the heterogeneity in expectations and study its implications for policy. In this paper, we consider *home bias* as yet another potential source of heterogeneity in expectations. The idea is that agents, as they acquire information, focus more strongly on news about domestic variables than about foreign variables. As discussed in more detail below, we capture this home bias in expectations by a parameter that weighs down foreign variables in agents' information sets. We interpret this home bias parameter as a parsimonious modeling alternative to more elaborate micro-founded theories, such as the concept of rational inattention of Sims (2003) or the sparse maximization approach of Gabaix (2014). In Sims (2003), agents incur a fixed cost when acquiring and processing information, thereby rendering the use of all avail-

[3]Significant contributions on the stability properties of RPE models are Berardi (2007), Berardi and Duffy (2007), Branch and Evans (2006a) and Branch and Evans (2011).

able information sub-optimal. In Gabaix (2014), agents' inattention is source-dependent, meaning that some information is ignored (to some extent) if it is deemed unnecessary for the optimization problem at hand.

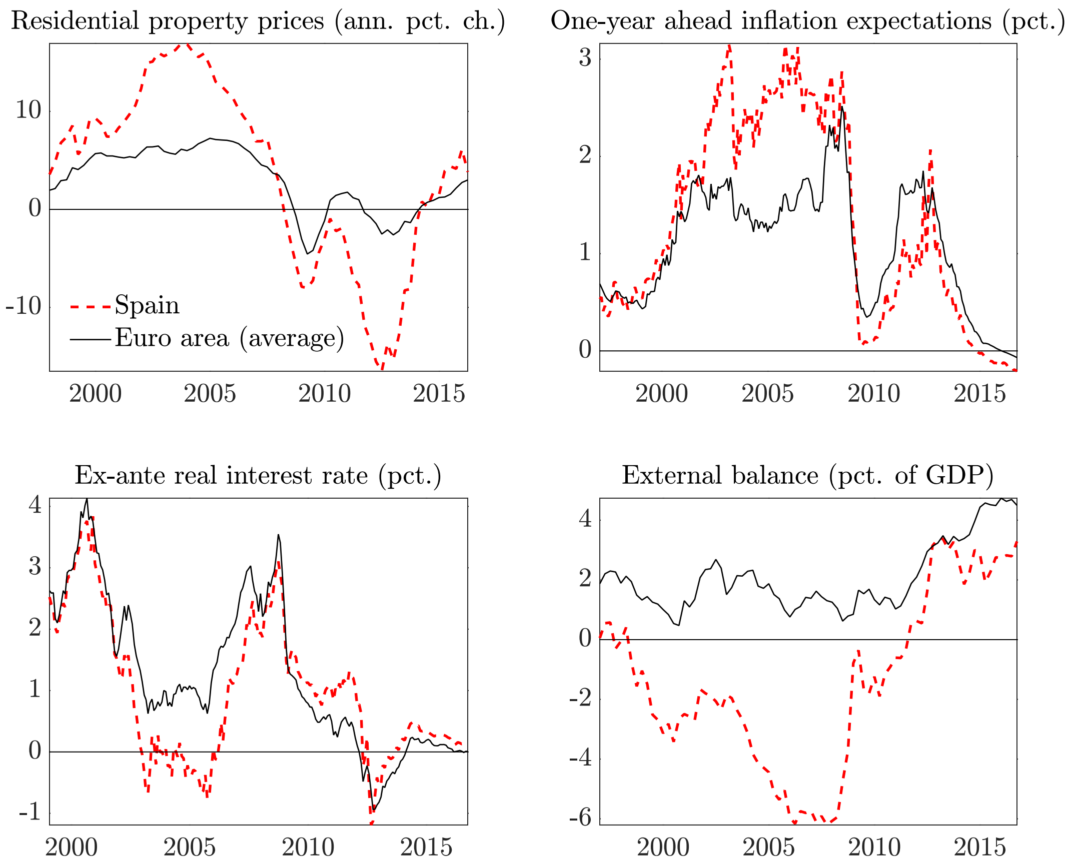
Home bias in the formation of expectations may arise for several reasons. First, as in Sims (2003), but also Simon (1984) and more recently Coibion et al. (2015), the capacity to acquire and process information may be limited, especially with regards foreign information due to language barriers, unfamiliarity with certain data sources, uncertainty about data quality, the importance of trade linkages, etc. For these reasons, it might also take more time to process information of a foreign rather than a domestic nature, which increases the inertia in agents' expectations about foreign economic conditions.[4] Second, expectations could be(come) home biased in times of large country-specific shocks during which agents are either overly optimistic or pessimistic and exhibit myopic behavior. For instance, during a housing boom, agents may be inclined to focus more on domestic economic conditions rather than on foreign conditions, since the former is less likely to be affected by the latter in times of domestically driven buoyancy.

In this section, we offer some empirical evidence for home bias in expectations based on a case study of Spanish inflation expectations during the Spanish housing boom. The top-left panel of Figure 2 shows the remarkable rise in residential property prices in Spain between 2003 and 2006 (dashed line), which deviated strongly from average property prices in the euro area (solid line). During this time, Spanish inflation expectations increased sharply and were well above the euro area average (top-right panel), resulting in a below average real interest rate (lower-left panel). What is further apparent is that the divergent dynamics of Spanish housing prices, inflation expectations and the real interest rate vis-à-vis the rest of the euro area coincided with a deepening of Spain's trade deficit (lower-right panel).

To see whether Spanish inflation expectations were more sensitive to domestic variables

[4]In addition, Van Nieuwerburgh and Veldkamp (2009), in explaining the home bias puzzle in equity portfolios, show that investors may choose to learn more about home assets than about foreign assets in order to magnify their comparative advantage in home assets, thereby achieving higher excess returns.

Figure 2: The Spanish housing boom



Note: The ex-ante real interest rate is calculated as the difference between the one-year nominal EURIBOR and one-year ahead inflation expectations. *Source*: ECB Statistical Data Warehouse (residential property price indices), European Commission's Consumer Survey (inflation expectations), Datastream (EURIBOR), OECD Quarterly National Accounts dataset (external balance).

than to foreign variables during the housing boom, we regress one-year ahead inflation expectations, conditional on information available at quarter t, i.e. $\pi^e_{t+4|t}$, on a constant and lagged annual changes in the GDP deflator, π_{t-1}, and the real exchange rate, Δq_{t-1}. The latter two regressors are proxies for the available information on, respectively, domestic and foreign price changes. To track the contributions of domestic and foreign variables to Spanish inflation expectations over time, we estimate the following model with time-varying parameters:

$$\begin{aligned}\pi^e_{t+4|t} &= \beta_{0,t} + \beta_{1,t}\pi_{t-1} + \beta_{2,t}\Delta q_{t-1} + e_t \quad (1)\\ &= x_t\beta_t' + e_t, \quad e_t \sim \mathcal{N}(0, R),\\ \beta_t &= \beta_{t-1} + v_t, \quad v_t \sim \mathcal{N}(0, Q), \quad (2)\end{aligned}$$

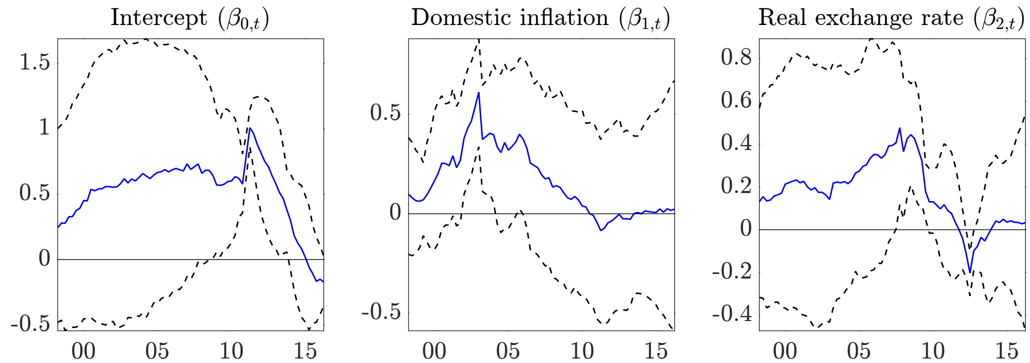
with $\text{cov}(e_t, v_t) = 0$ and $x_t \equiv [1,\ \pi_{t-1},\ \Delta q_{t-1}]'$, and where $\beta_t \equiv [\beta_{0,t},\ \beta_{1,t},\ \beta_{2,t}]'$ are the time-varying parameters to be estimated. We use Bayesian estimation techniques and apply a Kalman filter with a diffuse prior to estimate the model (1)-(2).[5] Quarterly data for the GDP deflator and the real exchange rate, from 1998Q2 to 2016Q2, were obtained from Eurostat. The real exchange rate measures the trade-weighted change in relative unit labor costs vis-à-vis eighteen trading partners within the euro area.[6] For inflation expectations, we used quarterly averages of the monthly one-year ahead inflation expectations extracted from the European Commission's Consumer Survey.

Figure 3 shows the estimates for the time-varying coefficients in Equation (1). According to the figure, the contribution of domestic inflation to the dynamics of Spanish inflation expectations, as governed by $\beta_{1,t}$, increased during the Spanish housing boom (see middle

[5] The prior distribution for the variance R is an Inverse Gamma distribution, i.e. $R \sim IG(T_0/2, D_0/2)$, with the degrees of freedom and scaling parameter initialized at $T_0 = 1$ and $D_0 = 0.1$. The prior for Q is an Inverse Wishart distribution, i.e. $Q \sim IW(Q_0, T_0)$, where Q_0 is initialized as the identity matrix. Remaining priors are $\beta_t = 0$, $R = 1$ and $Q = I_3$. A total of 12,000 draws were used for the Gibbs sampling algorithm, of which the first 10,000 draws were discarded. See Primiceri (2005) for further details on the Bayesian estimation of time-varying parameter models.

[6] Using the real effective exchange rate based on relative consumer prices yields similar results.

Figure 3: Time-varying contributions to Spanish inflation expectations



Note: The figure shows estimates for the time-varying coefficients in Equation (1). Solid (dashed) lines reflect the 50th (16th and 84th) percentiles from the posterior distribution.

panel). During the Great Recession, when euro area countries faced a common shock, the dynamics of Spanish inflation expectations were no longer driven by domestic inflation. Furthermore, there is little evidence that, during the housing boom, inflation expectations were influenced by changes in the real exchange rate (right panel). These results thus support our conjecture that changes in domestic variables are more important for the formation of (inflation) expectations than changes in foreign variables, at least in times when an economy faces large country-specific shocks.

3 The model

For our main analysis, we apply a New Keynesian model for a monetary union that consists of two countries, Home (H) and Foreign (F). Since the model is, by now, quite standard, we defer a detailed description to the Online Appendix. Here, we provide a brief overview of the model's main features.

Each country is populated by a continuum of households. Households choose consumption and labor supply, while aiming to maximize expected life-time utility subject to appropriate budget constraints. Final consumption goods consist of bundles of differentiated intermediate goods, which are traded internationally and produced by monopolistic wholesale firms using a labor-intensive constant returns to scale production technology. We assume labor is completely immobile across countries, yet perfectly mobile within countries. Intermediate goods firms are subject to a price-setting constraint à-la Calvo (1983). All firms are owned by the households of the country in which the firm is domiciled. We assume that expectations are the same across households and firms within the same country. However, due to home bias, we allow expectations to be different *across* countries. Asset markets are complete, which implies that households can trade a complete set of state-contingent Arrow-Debreu assets. Nevertheless, due to the cross-country heterogeneity in expectations, perfect international risk sharing breaks down.[7] Finally, a common central bank sets monetary policy and targets union-wide aggregates.

In the sequel of this section, we discuss how, in our model, expectations are formed and provide details on how home bias in expectations is introduced. The section ends with a brief discussion on the model's implications for the persistence of country-specific shocks under monetary union and home biased expectations.

3.1 How expectations are formed

Our point of departure is the linearized version of the two-country New Keynesian model described above, which is presented in Appendix A. Let x_t be a vector of endogenous variables, realized at t, and v_t a vector of exogenous variables. The model's dynamics can then be summarized in canonical form by the following two equations:

$$Ax_t = \sum_{i=\{H,F\}} B^i \tilde{E}_t^i x_{t+1} + Cx_{t-1} + Dv_t, \tag{3}$$

$$v_t = \varrho v_{t-1} + \varepsilon_t, \tag{4}$$

where A, B^i, C and D are coefficient matrices of appropriate sizes, ϱ a diagonal matrix with all eigenvalues inside the unit circle, and ε_t a vector of i.i.d. normal shocks with mean zero and variance σ^2. $\tilde{E}_t^i$ is the expectations operator used by residents of country i.

[7] See Santoro (2017) for a more detailed description of the resulting competitive equilibrium.

The Rational Expectations Equilibrium (REE) of the model above has the following form:

$$x_t = \overline{\Lambda}_1 x_{t-1} + \overline{\Lambda}_2 v_t. \tag{5}$$

Under the adaptive learning approach considered here, agents do not know the true coefficients matrices, $\left\{\overline{\Lambda}_1, \overline{\Lambda}_2\right\}$, yet must obtain estimates using the data available to them (see Evans and Honkapohja, 2001).[8] Note that, if the number of state variables is smaller than the total number of endogenous variables, some columns of $\overline{\Lambda}_1$ would be empty. However, here we allow for the possibility that agents attach non-zero weights to lagged variables other than the model's state variables in the solution given by (5).

When expectations are home biased, domestic and non-domestic variables are weighted differently in agents' information sets. Specifically, let $x_{d,t}$ denote a subset of x_t containing only **d**omestic variables, and $x_{n,t}$ a subset containing **n**on-domestic variables. The data vector can then be partitioned as $x_t = [x_{d,t},\ x_{n,t}]'$. Similarly, partition the exogenous variables into domestic and non-domestic shocks, i.e. $v_t = [v_{d,t},\ v_{n,t}]'$. Using this transformation, we can write household i's *Perceived Law of Motion* (PLM) as follows:

$$x_t = \Lambda^i_{0,t-1} + \Lambda^i_{1,t-1} \begin{bmatrix} I & \mathbf{0} \\ \mathbf{0} & I\omega \end{bmatrix} \begin{bmatrix} x_{d,t-1} \\ x_{n,t-1} \end{bmatrix} + \Lambda^i_{2,t-1} \begin{bmatrix} I & \mathbf{0} \\ \mathbf{0} & I\omega \end{bmatrix} \begin{bmatrix} v_{d,t} \\ v_{n,t} \end{bmatrix}, \tag{6}$$

with $\mathbf{0}$ the zero matrix of conformable size, and where $\omega \in [0, 1]$ denotes the home bias parameter. For $\omega < 1$, non-domestic variables are given a lower weight than domestic variables and we say that expectations are home biased. For $\omega = 1$, there is no home bias in expectations and, because all agents use the same information set, expectations are the

[8] In this paper, we follow the so-called 'Euler equation learning' approach, which assumes agents make consumption decisions to satisfy one-period ahead perceived Euler equations. Consequently, agents do not take into account future revisions of their beliefs when forming expectations. See also Evans and Honkapohja (2001) and Bullard and Mitra (2002).

same across countries.[9,10]

Denote $\Lambda_t^i \equiv \left[\Lambda_{0,t}^i,\ \Lambda_{1,t}^i,\ \Lambda_{2,t}^i\right]$ and $z_t \equiv [1,\ x_{t-1},\ v_t]'$. Whenever new data becomes available, agents update their estimates for Λ_t^i using the following updating equations:

$$\Lambda_t^i = \Lambda_{t-1}^i + \gamma_t \left(M_t^i\right)^{-1} \tilde{z}_t^i \left(\tilde{x}_t^i - \Lambda_{t-1}^i \tilde{z}_t^i\right), \tag{7}$$

$$M_t^i = M_{t-1}^i + \gamma_t \left(\tilde{z}_t^i \left(\tilde{z}_t^i\right)' - M_{t-1}^i\right). \tag{8}$$

The tildes above the data vectors, $\tilde{x}_t^i$ and $\tilde{z}_t^i$, denote the *observed* data. Equation (7) shows that the extent by which the coefficients are updated depends on the size of the forecast error, $\tilde{x}_t^i - \Lambda_{t-1}^i \tilde{z}_t^i$. The weight on the forecast error is determined by M_t^i, i.e. the moment matrix of $\tilde{z}_t^i$, and the gain parameter γ_t, which controls the speed with which agents learn. In what follows, we assume a constant gain, i.e. $\gamma_t = \gamma$ for all t, such that agents discard old data from their sample and assign more weight to new information (as is done in rolling window regressions).[11] By limiting the amount of data used, the home bias parameter ω affects the moment matrix of the data, M_t^i, which in turn alters the way agents update their estimates of Λ_t^i by (7) and (8). Hence, agents' beliefs on economic dynamics are shaped by the degree of home bias in expectations.

[9]Our bias parameter ω can be related to Gabaix (2014)'s source-dependent attention vector in his sparse max model, in which agents, when making decisions, pay less or no attention to some variables in the model. A crucial difference with our approach is that, in the sparse max model, the attention vector is chosen optimally so as to minimize the costs of inattention, whereas we treat the attention bias as constant.

[10]The bias parameter ω could also be interpreted as reflecting the proportion of agents that focus solely on domestic variables while forming expectations. These agents may, for instance, earn their income from the production of non-tradable goods, which leaves them less concerned about foreign economic developments. In that case, the aggregate data vector observed in country i is simply a weighted average of the information sets stemming from the tradable sector, $x_{T,t}^i$, and the non-tradable sector, $x_{NT,t}^i$, i.e.

$$\begin{aligned}
\tilde{x}_t^i &= \omega x_{T,t}^{i'} + (1-\omega)\, x_{NT,t}^{i'} \\
&= \omega \begin{bmatrix} x_{d,t-1} \\ x_{n,t-1} \end{bmatrix}' + (1-\omega) \begin{bmatrix} x_{d,t-1} \\ \mathbf{0} \end{bmatrix}' \\
&= \begin{bmatrix} I & \mathbf{0} \\ \mathbf{0} & I\omega \end{bmatrix} \begin{bmatrix} x_{d,t-1} \\ x_{n,t-1} \end{bmatrix}.
\end{aligned}$$

[11]As discussed in Branch et al. (2014), adaptive learning models that use a constant gain perform better in terms of empirical fit than models using a time-varying gain, which in turn are typically used to study asymptotic properties and convergence.

In Section 2, we offered a number of reasons for why agents may be home biased when acquiring and processing new information. In the model, we circumvent the task of building the micro-foundations that explain such behavior, and instead propose a more parsimonious approach to control the degree of home bias in expectations by a single parameter, ω. Thereby, we can easily isolate the effects of home bias in expectations by considering different values for ω. Moreover, this approach allows us to obtain, quite straightforwardly, empirical estimates for ω by confronting the adaptive learning model (6)-(8) to the data. We return to this matter in Section 4.

At this point, it is important to point out how the home bias parameter affects the estimated coefficients and, thereby, the dynamics of the system. What is crucial to understand is that, due to the self-referential nature of the system in which realized variables and expectations affect each other, the regressors in the PLM (6) are endogenous and so the Frisch-Waugh-Lovell theorem does not apply here. Hence, one cannot simply rewrite the updating equations (7)-(8) in such a way that the effect of the home bias parameter on the estimated coefficients is nullified.

To see this more formally, consider a variant of the Cobweb model in which the evolution of the price level, p_t, depends on expected prices, past prices and some some exogenous variable, v_t:

$$p_t = aE_t p_{t+1} + bp_{t-1} + cv_t,$$

with a, b and c parameters, and where v_t evolves according to $v_t = \rho v_{t-1} + \varepsilon_t$, with $\varepsilon_t \sim \mathcal{N}(0, \sigma^2)$ and $|\rho| \in [0, 1]$. Assume that agents use the following forecasting equation as their PLM:

$$p_t = \alpha p_{t-1} + \beta v_t.$$

Updating this equation by one period and substituting the result for $E_t p_{t+1}$ in the Cobweb model yields the so-called *Actual Law of Motion* (ALM):

$$p_t = \left(a\alpha^2 + b\right) p_{t-1} + \left[a\beta\left(\alpha + \rho\right) + c\right] v_t.$$

The rational expectations solution to this model is given by

$$\overline{\alpha}_{REE} = \frac{1 - \sqrt{1 - 4ab}}{2a}, \qquad \overline{\beta}_{REE} = \frac{c}{1 - a\left(\overline{\alpha}_{REE} + \rho\right)}.$$

Now suppose that, as in our model, both the endogenous and exogenous variables enter the PLM with some degree of imprecision, the latter being captured by the parameter ω:

$$p_t = \alpha\omega p_{t-1} + \beta\omega v_t.$$

The fixed point of the function that maps the PLM to the ALM is now given by[12]

$$\overline{\alpha}_{RPE} = \frac{1 - \sqrt{1 - 4ab\omega^2}}{2a\omega^2}, \qquad \overline{\beta}_{RPE} = \frac{c}{1 - a\omega\left(\overline{\alpha}_{RPE}\omega + \rho\right)}.$$

Due to the bias in the observability of the endogenous variables, the system saddles down to an RPE, rather than the REE. Thus, the bias parameter ω alters the estimated coefficients, and thereby the dynamics of the model, in a non-trivial way. If, on the other hand, the variables would be perceived without bias, i.e. $\omega = 1$, one recovers the REE. Moreover, and as shown in the Online Appendix, if the bias would apply only to the exogenous variables, one could apply the Frisch-Waugh-Lovell theorem and recover the REE as well. Hence, it is because of the self-referential nature of the model *and* because the bias in perception applies to the endogenous variable that the system departs from the REE.

[12] See the Online Appendix for a brief derivation, as well as a more extensive discussion on the implications of biased perceptions in adaptive learning models.

3.2 Monetary union and the persistence of asymmetric shocks

Before we study the implications of the home bias in expectations for macroeconomic imbalances under monetary union, which is the main focus of the paper, we first show how being in a monetary union may cause country-specific shocks be more amplified, even under rational expectations.

Consider first the interest rate rule that governs the single central bank's monetary policy:

$$\frac{R_t}{R} = \left(\frac{\pi_{MU,t}}{\pi_{MU}}\right)^{\phi_\pi} \left(\frac{y_{MU,t}}{y_{MU}}\right)^{\phi_y},$$

with $\phi_\pi > 1$ and $\phi_y \geq 0$ monetary policy parameters. This rule relates the nominal interest rate R_t to deviations of union-wide inflation and output, $\pi_{MU,t}$ and $y_{MU,t}$, from their respective steady-state values. Because the central bank only targets union-wide aggregates, country-specific shocks may not always prompt a sufficiently strong monetary response insofar union-wide aggregates remain largely unchanged. This might be the case, for instance, if shocks are negatively correlated across countries or borne from relatively small regions within the union. It is this inability to tailor monetary policy to country-specific disturbances that allows temporary shocks to have long-lasting effects.

To better understand the peculiar nature of the propagation of shocks under monetary union, consider the linear version of the Home Euler equation (see Appendix A):

$$\sigma \hat{c}_t^H = \sigma \tilde{E}_t^H \hat{c}_{t+1}^H - \left(\hat{R}_t - \tilde{E}_t^H \hat{\pi}_{H,t+1}^H\right) + \hat{\alpha}^H \left(\tilde{E}_t^H \hat{q}_{t+1} - \hat{q}_t\right) + (1 - \rho_D)\, z_{D,t}^H,$$

where $\hat{c}_t^H$ denotes household consumption, $\hat{\pi}_{H,t}^H$ domestic inflation, and $\hat{q}_t$ the real exchange rate, all expressed in percentage deviations from steady state. $z_{D,t}^H$ is a demand shock that evolves according to an AR(1) process with auto-correlation coefficient $\rho_D \in [0, 1]$. The key parameters that govern consumption dynamics are $\sigma > 0$, which measures the inverse of the elasticity of intertemporal substitution, and $\hat{\alpha}^H \geq 0$, which determines country openness.

A positive shock to Home demand, i.e. an increase in $z^H_{D,t}$, raises consumption and, hence, firm production. Consequently, real marginal costs go up which, by the optimal price-setting condition of the firm, leads to higher expected domestic inflation, $\tilde{E}^H_t \hat{\pi}^H_{H,t+1}$. Higher expected inflation reduces the ex-ante real interest rate, $\hat{R}_t - \tilde{E}^H_t \hat{\pi}^H_{H,t+1}$, which prompts a further increase in household consumption and inflation. We refer to this pro-cyclical channel as the *real interest rate channel*. Under monetary union, this channel can be quite strong, even under rational expectations, since the central bank does not target the nominal interest rate, $\hat{R}_t$, to stabilize inflation in a specific region of the union. Rather, the central bank is concerned only with inflation at the union-wide level. Therefore, even if short-lived, country-specific shocks can have strong and persistent effects on the economy.

The rise in inflation, however, also leads to an expected appreciation of the real exchange rate–i.e. $\hat{q}_t$ is expected to fall. Due to its negative effect on net exports and aggregate domestic income, a real exchange rate appreciation reduces consumption. This counter-cyclical channel is referred to as the *real exchange rate channel*, and its strength is increasing in the degree of country openness, measured by $\hat{\alpha}^H$. Particularly, the more integrated are international goods markets, the more sensitive are net exports (and thereby aggregate income and consumption) to changes in the real exchange rate, and therefore the stronger the real exchange rate channel.

Note that the strength of the real interest rate channel relative to the real exchange rate channel also depends on the degree of home bias in expectations. If the home bias is strong, expectations do not respond much to changes in the real exchange rate, since the real exchange rate is a non-domestic variable and agents ignore such variables, at least to some extent. As a consequence, agents do not fully internalize real exchange rate fluctuations when making consumption decisions, which weakens the real exchange rate channel. Thus, by strengthening the real interest rate channel relative to the real exchange rate channel, the home bias in expectations has the potential to amplify country-specific disturbances, over and above the amplification that potentially arises already from the central bank's mandate of targeting union-wide aggregates.

4 The effects of an asymmetric demand shock

To illustrate the relationship between home biased expectations and macroeconomic imbalances, we study the effects of a country-specific, and hence asymmetric, demand shock. The reason for focusing on asymmetric, rather than symmetric, shocks is twofold. First, given that the common central bank only has one instrument to target union-wide aggregates, i.e. the nominal interest rate, monetary policy is unable to optimally respond to country-specific shocks, as any centralized monetary action would necessarily entail unwanted outcomes in some parts of the monetary union. Second, as mentioned in Section 2, expectations are more likely to be(come) home biased in the event of large country-specific shocks, to the extent these cause agents to view domestic economic conditions as being detached from the foreign world. If, on the other hand, member countries were confronted by the same shock, then monetary policy would be better able to absorb the shock, and prevent divergent economic dynamics, which would also help anchor expectations. Therefore, in what follows, we consider demand shocks that are perfectly negatively correlated across two equally-sized countries, such that, following such shocks, union-wide aggregates and monetary policy are left unchanged.

To obtain impulse response functions, we follow the approach suggested by Eusepi and Preston (2011). In particular, we simulate the model twice, whereby in each round the economy is exposed to a series of small uncorrelated shocks, randomly drawn from a Normal distribution, for T periods, during which agents periodically update their beliefs according to the learning algorithm described in Section 3. In following with the literature, we assume that, at the start of each simulation round, agents' beliefs on how the economy works coincide with those under rational expectations. In one of the two simulations, the economy is hit by the shock of interest at period $T - K$, where K is the impulse response horizon. The impulse responses are then calculated as the difference between the two simulation rounds from period $T - K$ to T. This approach helps track the response of the economy to the asymmetric demand shocks while agents are still updating their beliefs. We set $T = 500$.

4.1 Estimates for the home bias parameter

In order to get a sense of empirically plausible values for the home bias parameter, ω, and the gain parameter, γ, we confront the learning model (6)-(8) to data on household inflation expectations in the euro area. Specifically, as in Section 2, we conjecture that one-year ahead inflation expectations in country i, $\tilde{E}_t^i \pi_{t+4}^i$, are formed based on lagged annual changes in the GDP deflator, $\pi_{i,t-1}^i$, and the real exchange rate, Δq_{t-1}^i:

$$\tilde{E}_t^i \pi_{t+4}^i = \Lambda_{0,t-1}^i + \Lambda_{1,t-1}^i \begin{bmatrix} 1 & 0 \\ 0 & \omega \end{bmatrix} \begin{bmatrix} \pi_{i,t-1}^i \\ \Delta q_{t-1}^i \end{bmatrix}.$$

Consistent with the learning model presented in Section 3, the degree of home bias in expectations is captured by the parameter ω. This parameter captures the possibility that the real exchange rate, whose fluctuations depend on foreign price developments, is partially ignored by households. In each period t, the coefficient matrices $\Lambda_{0,t-1}^i$ and $\Lambda_{1,t-1}^i$ are updated using Equations (7) and (8), for $\tilde{x}_t^i \equiv \left[\pi_{i,t-1}^i,\ \omega \Delta q_{t-1}^i\right]'$, by evaluating households' inflation expectations against actual CPI inflation. All data are taken from Eurostat and cover the period 1998Q1 to 2016Q2. We also consider a shorter sample that excludes the crisis years. The simulations are performed many times, each time using different values for $\omega \in [0, 1]$ and $\gamma \in [0, 1]$, and compared against actual inflation expectations extracted from the European Commission's Consumer Survey, $\pi_{i,t+4|t}^e$. Particularly, we calculate the Mean Square Comparison Error, $MSCE^i$:

$$MSCE^i = \frac{1}{T} \sum_{t=1}^{T} \left(\pi_{i,t+4|t}^e - \tilde{E}_t^i \pi_{t+4}^i \right)^2,$$

Table 1: Estimates for the home bias and gain parameters in the euro area

	1998Q1-2016Q2		1998Q1-2008Q2	
Country	*Home bias, ω*	*Gain, γ*	*Home bias, ω*	*Gain, γ*
Belgium	0.47	0.033	0.35	0.041
Germany	0.3	0.083	0.17	0.07
Ireland	0	0.005	0	0.02
Greece	0.16	0.043	0	0.066
Spain	0.29	0.008	0	0.008
France	0	0.018	0	0.019
Italy	1	0.007	0	0.01
Netherlands	0.34	0.021	0.85	0.025
Austria	0.66	0.02	0	0.022
Portugal	0	0.046	0	0.023
Finland	0	0.004	0	0.007

where t ranges from 1998Q1 to 2016Q2, and report the home bias and gain parameters that yield the lowest $MSCE^i$.[13,14]

The results are reported in Table 1. The second column shows the estimates for the home bias parameter ω for the full sample period and suggests that there are large differences in the estimated degree of home bias in expectations across countries. In some countries, such as Austria, Belgium and Italy, expectations are best captured when using a relatively large value for ω, implying that consumers internalize, at least to some extent, foreign price developments when forming their inflation expectations. However, other countries, like Finland, France, Ireland and Portugal, exhibit a strong degree of home bias in expectations, as expectations are best explained if ω is set to zero. The estimates for ω for the shorter sample, shown in the fourth column, are generally smaller than those found using the full sample. The shorter sample, by not including the crisis years, is dominated by episodes of local, rather than common, shocks, which may explain the stronger degree of home bias in expectations. These results are in line with those from the case study in Section 2, where we observed an increase in the degree of home bias in Spanish inflation expectations during the

[13] For each simulation, we initialize the coefficient and moment matrices by setting them equal to zero and the identity matrix, respectively. These are used to obtain inflation forecasts for the first four quarters.

[14] In the Online Appendix, we plot the simulated inflation forecasts associated with the lowest $MSCE^i$ against actual inflation expectations.

Spanish housing boom and a fall in home bias in the years thereafter. Given the dispersion in our estimates, we consider a wide range of values for ω in the analysis below.

With regards the gain parameter, we set γ equal to 0.01. This value lies within our own estimates for γ and in between existing empirical findings, with somewhat higher estimates reported for professional forecasters (Branch and Evans, 2006b) and lower estimates for households (Pfajfar and Santoro, 2010).

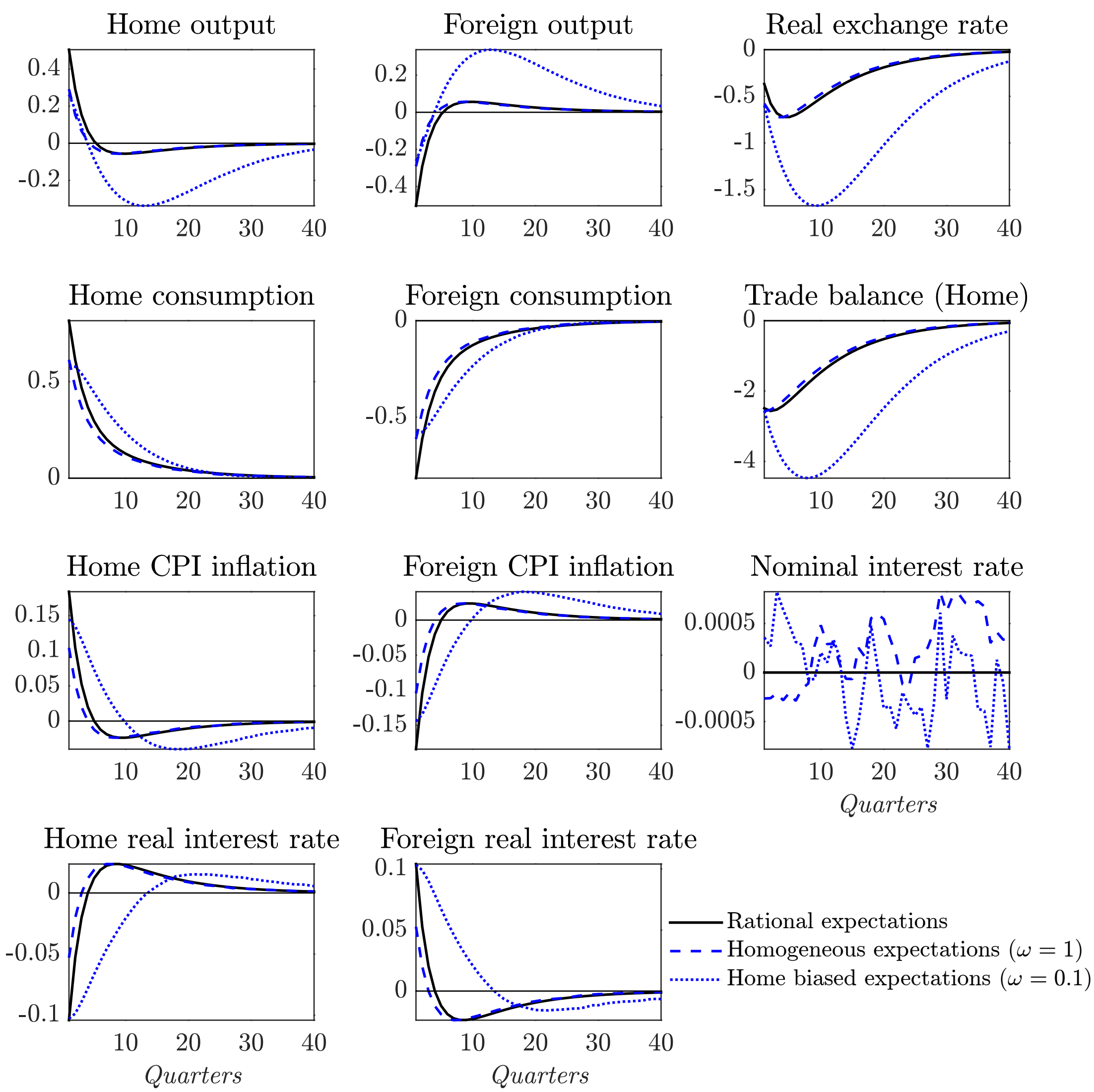
The remaining model parameters are calibrated at a quarterly frequency. An overview of the benchmark calibration can be found in Table 2 in Appendix A. Many parameters are assigned values that are common in the literature. The parameter that governs country openness, α, is assigned a somewhat lower value than what is typically used, as we wish to remain consistent with the idea of expectations being home biased. Specifically, we set $\alpha = 0.25$, which matches the average import-to-output ratio of bigger Euro area countries like France, Italy or Spain in recent years. We do, however, experiment with alternative values for α in the analysis below.

4.2 Impulse response functions

The responses of the endogenous variables to an asymmetric demand shock are displayed in Figure 4. The solid lines are the responses under rational expectations, the dashed lines the responses under adaptive expectations without home bias, and the dotted lines the responses under adaptive expectations with home bias. The shock is assumed to be positive in Home and negative in Foreign. Therefore, consumption rises in Home, which leads to an increase in Home output and inflation, whereas Foreign consumption, output and inflation fall.

Since the shock has opposing effects across countries, union-wide inflation and output remain unchanged and so the short-term nominal interest rate set by the central bank does not respond (much) to the shock. Consequently, the real interest rate falls in Home, which further stimulates the initial demand shock and the consequent rise in Home consumption and inflation. As discussed in Section 3.2, this pro-cyclical real interest rate channel arises from

Figure 4: Responses to an asymmetric demand shock



Note: Responses are expressed in percentage deviations from steady state. The demand shock is perfectly negatively correlated across the two countries, with Home (Foreign) experiencing a positive (negative) shock.

the household's Euler equation that links expected consumption growth to the ex-ante real interest rate. In Foreign, the opposite occurs and economic activity is suppressed by a fall in Foreign inflation (expectations) and an increase in the real interest rate. Thus, even under rational expectations, the asymmetric demand shock generates persistent macroeconomic imbalances within the monetary union, due to the real interest rate channel.

Meanwhile, the rise in Home inflation leads to an appreciation of the real exchange rate, which dampens the economic boom through a fall in the trade balance and a contraction in aggregate demand. Conversely, the real exchange rate depreciates from the perspective of Foreign, which helps recover economic activity. Hence, the destabilizing real interest rate channel is offset, endogenously, through a stabilizing real exchange rate channel that aids in restoring the trade imbalance within the monetary union.

The change in the real exchange rate induces firms in Home to lower their prices in order to remain competitive in international goods markets. This effect can be inferred from the linearized price-setting condition for intermediate goods firms in Home:

$$\hat{\pi}^H_{H,t} = \beta \tilde{E}^H_t \hat{\pi}^H_{H,t+1} + \kappa \left(\varphi \hat{y}^H_t + \sigma \hat{c}^H_t + \hat{\alpha}^H \hat{q}_t \right),$$

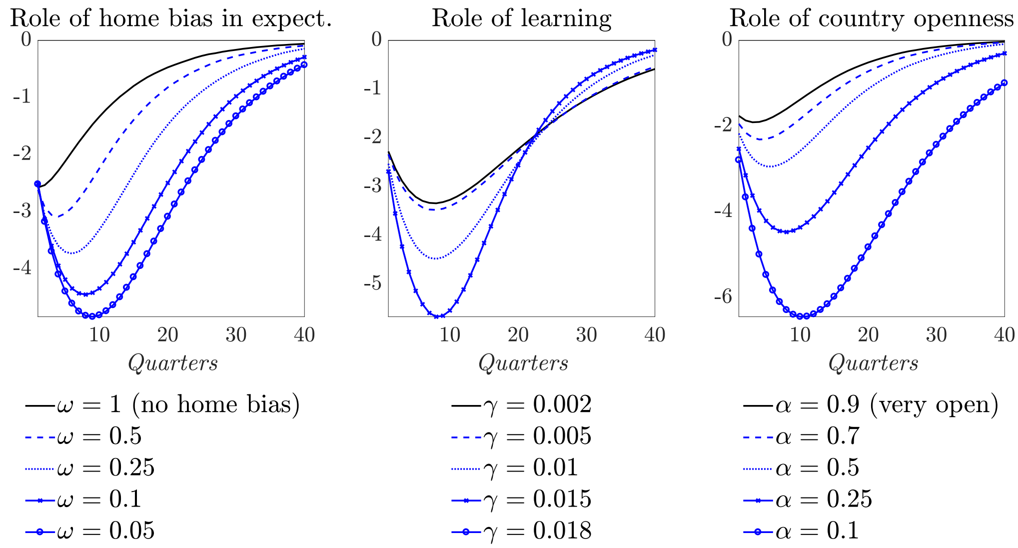
with $\hat{y}^H_t$ Home output, $\beta \in (0,1)$ the household's discount rate, $\varphi > 0$ the inverse of the Frisch elasticity of labor supply and where $\kappa > 0$ is determined by the probability of non-price adjustment. An appreciation of the real exchange rate, i.e. a fall in $\hat{q}_t$, reduces real marginal costs through a decline in aggregate demand, which causes firms to adjust prices downwards and domestic inflation, $\hat{\pi}^H_{H,t}$, to fall. Foreign firms behave in the opposite direction. This nominal price adjustment counteracts the effects of the real interest rate channel such that, over time, the real interest rate rises in Home and falls in Foreign, until the regional dispersion in economic activity disappears.

Note that the strength of this adjustment mechanism depends on the degree of country openness and price stickiness. The more open the economy is to foreign trade, i.e. the higher is $\hat{\alpha}^H$, the more sensitive are price-setting decisions and aggregate demand to changes in the real exchange rate. Moreover, the more flexible are prices, i.e. the higher is κ, the faster do prices adjust for a given change in the real exchange rate. In both cases, macroeconomic convergence results more rapidly following an asymmetric demand shock.

Under home biased expectations, the responses of the endogenous variables are more pronounced and more persistent than under rational and unbiased expectations. In generating the impulse responses under home biased expectations, we set ω equal to 0.1. Other values for ω are considered below. Due to the home bias, agents do not fully internalize the relative change in domestic versus foreign prices and, as a consequence, underestimate the effects of the real exchange rate on the domestic economy. The home bias in expectations thus weakens the real exchange rate channel. Moreover, as agents focus more on domestic economic conditions, inflation expectations in Home (Foreign) remain high (low), and the real interest rate low (high), for a much longer period than in the absence of home bias in expectations, which strengthens the real interest rate channel. Due to the shift in the relative strengths of the two channels, the demand shock has a more amplified effect under home biased expectations than if expectations were rational or free from any home bias. This is reflected by a sharper adjustment of the real exchange rate and larger macroeconomic imbalances following the demand shock.

The main message conveyed by these impulse responses is that real interest rate differentials across member states of a monetary union can lead to larger and more prolonged macroeconomic imbalances if agents, while forming expectations, rely mostly on information inferred from domestic variables, while ignoring foreign and union-wide variables. The amplification of the effects of real interest rate differentials is due to a pro-cyclical real interest rate channel that undercuts a stabilizing real exchange rate channel. To overcome the home bias in agents' expectations, a more extreme adjustment in nominal prices is required, given that, under monetary union, the nominal exchange rate is fixed and monetary policy tools are powerless (or at least inefficient) in the face of asymmetric shocks.

Figure 5: Home's trade balance response to an asymmetric demand shock



Note: $\omega \in [0, 1]$ measures the home bias in expectations, $\gamma > 0$ is the gain parameter in Equations (7) and (8), and $\alpha \in [0, 1]$ determines the import-share (and thereby country openness) in household consumption.

In the left panel of Figure 5, we analyze further the implications of home bias in expectations for macroeconomic imbalances. We use Home's trade balance as a measure of macroeconomic imbalances and focus on its response to an asymmetric demand shock under different values for the home bias parameter ω. The results suggest that, for a greater degree of home bias in expectations, i.e. for lower values of ω, macroeconomic imbalances are larger and macroeconomic adjustment is slower. The intuition follows form our discussion above: the home bias causes agents to ignore relative international price changes and underestimate the impact of the real exchange rate on economic dynamics, which weakens the real exchange rate channel and strengthens the real interest rate channel. These results are in line with the empirically observed positive relationship between cross-country dispersion in inflation expectations and external balances in the euro area, as shown in the bottom panel of Figure 1 in the Introduction: the stronger is the home bias in expectations, the more dispersed are expectations across countries and the greater are trade imbalances once countries face asymmetric shocks.

In the middle panel of Figure 5 we plot the response of Home's trade balance to an asymmetric demand shock for different values of the gain parameter, γ, while keeping the home bias parameter fixed at $\omega = 0.1$. As is evident from the figure, the larger is the gain parameter, the greater is the resulting macroeconomic imbalance within the monetary union for a given asymmetric demand shock. In particular, a higher gain implies that agents assign a greater weight to current shocks when updating their beliefs. To the extent these beliefs are home biased, shocks are amplified the higher is γ, as agents respond more strongly to *domestic* shocks. However, a higher gain also implies that agents infer more quickly the stabilizing role of the real exchange rate, thus inducing a more rapid macroeconomic adjustment.

Finally, in the right panel of Figure 5, the response of Home's trade balance to an asymmetric demand shock is generated under different assumptions about country openness, which is governed by α, while keeping the home bias parameter constant at $\omega = 0.1$. The results from this figure convey the following message: the more the economy is exposed to foreign trade, i.e. the higher is α, the stronger is the effect of the real exchange rate on economic conditions and therefore the quicker are agents to discover the counteracting forces of the real exchange rate. This result can also be interpreted differently by noting that consumer prices are a weighted average of both domestic and foreign producer prices. Evidently, the greater is the import share in household consumption, the more CPI inflation contains information on foreign price developments. Thus, even if foreign prices are largely ignored directly by agents, they are indirectly inferred through CPI inflation, and more so the more open is the economy to foreign trade. Consequently, despite the presence of home bias in *expectations*, large and prolonged macroeconomic imbalances following an asymmetric demand shock can be avoided if home bias in *consumption* is limited.

5 Stability and implications for monetary policy

In this section, which consists of two parts, we investigate how the stability of the monetary union depends on the home bias in expectations and what this implies for monetary policy. In the first part, we derive the conditions for E-stability of the model. We focus on the notion of *strong* E-stability in which agents, when making their forecasts, include both state and control variables in the data vector x_t and, therefore, are likely to over-parameterize their forecasting model. Furthermore, we use the concept of decreasing (rather than constant) gain learning, as it allows us to consider the possibility of convergent beliefs. In the second part, we perform numerical simulations to illustrate how the requirements for monetary policy to guarantee E-stability change under different degrees of home bias in expectations. We use the insights from the stability analysis to explain how monetary policy can mitigate the pro-cyclical effects of the real interest rate channel and promote macroeconomic stability.

5.1 The Restricted Perception Equilibrium

To derive the requirements for E-stability, we start by inserting agents' PLM (6), iterated one period ahead, into the state-space representation of the model, given by (3), after multiplying by A^{-1} from the left, to find the ALM:

$$\begin{aligned} x_t &= \left[\sum_{j=H,F} F^j \left(I + \Lambda_1^j \Omega_x^j \right) \Lambda_0^j, \sum_{j=H,F} F^j \left(\Lambda_1^j \Omega_x^j \right)^2 + G, \sum_{j=H,F} F^j \left(\Lambda_1^j \Omega_x^j \Lambda_2^j + \Lambda_2^j \varrho \right) \Omega_v^j + H \right] \begin{bmatrix} 1 \\ x_{t-1} \\ v_t \end{bmatrix} \\ &\equiv T\left(\Lambda^H, \Lambda^F\right) z_t, \end{aligned} \tag{9}$$

where $\Omega_k^i = \begin{bmatrix} I & \mathbf{0} \\ \mathbf{0} & I\omega \end{bmatrix}$, with $i = \{H, F\}$ and $k = \{x, v\}$, are matrices of conformable sizes that determine by how much the non-domestic variables and shocks are weighted down in agents' PLM.

The assumption of a potential home bias in agents' expectations features a crucial departure from the standard adaptive learning literature, namely that agents in country i observe only an imperfect version of the ALM, which is equal to Equation (9) multiplied by Ω_x^i. Accordingly, agents face the following mapping from their PLM to their *observed* ALM:

$$T^i\left(\Lambda_0^i,\ \Lambda_2^i\Omega_x^i,\ \Lambda_2^i\Omega_v^i\right) = \begin{bmatrix} \Omega_x^i\left[\sum_{j=H,F} F^j\left(I+\Lambda_1^j\Omega_x^j\right)\Lambda_0^j\right] \\ \Omega_x^i\left[\sum_{j=H,F} F^j\left(\Lambda_1^j\Omega_x^j\right)^2+G\right] \\ \Omega_x^i\left[\sum_{j=H,F} F^j\left(\Lambda_1^j\Omega_x^j\Lambda_2^j+\Lambda_2^j\varrho\right)\Omega_v^j+H\right] \end{bmatrix}'. \tag{10}$$

If domestic and non-domestic variables are weighted differently in agents' information sets, i.e. if $\omega < 1$, then the coefficients of the PLMs across countries are mapped into different observed ALMs, which implies that, in general, $\Lambda^H \neq \Lambda^F$. As a result, expectations differ across countries in steady state which prevents the economy from converging towards the REE. Instead, the economy saddles on an RPE (see Evans and Honkapohja, Chapters 3.6 and 13). A necessary condition for the existence of such an RPE is that the mapping function, $T^i\left(\Lambda^i\right)$, has a fixed point, i.e. $\hat{\Lambda}^i = \left[\hat{\Lambda}_0^i,\ \hat{\Lambda}_1^i,\ \hat{\Lambda}_2^i\right]$, satisfying

$$\hat{\Lambda}_0^i = \Omega_x^i\left[\sum_{j=H,F} F^j\left(I+\hat{\Lambda}_1^j\Omega_x^j\right)\hat{\Lambda}_0^j\right], \tag{11}$$

$$\hat{\Lambda}_1^i\Omega_x^i = \Omega_x^i\left[\sum_{j=H,F} F^j\left(\hat{\Lambda}_1^j\Omega_x^j\right)^2+G\right], \tag{12}$$

$$\hat{\Lambda}_2^i\Omega_v^i = \Omega_x^i\left[\sum_{j=H,F} F^j\left(\hat{\Lambda}_1^j\Omega_x^j\hat{\Lambda}_2^j+\hat{\Lambda}_2^j\varrho\right)\Omega_v^j+H\right]. \tag{13}$$

Because Equation (12) includes a quadratic matrix expression, multiple solutions for $\hat{\Lambda}_1^i$ may exist. On the other hand, $\hat{\Lambda}_2^i$ is uniquely pinned down by Equation (13) for any given $\hat{\Lambda}_1^i$. Further, note that the only solution for $\hat{\Lambda}_0^i$ is the zero vector.

The relevant ordinary differential equations that govern the E-stability properties of any

RPE are derived in Appendix B and given by

$$\frac{d\Lambda^i}{d\tau} = \varpi^i \left[T^i \left(\Lambda_{t-1}^H, \Lambda_{t-1}^F \right) \left(\Omega_x^i \right)' - \left(\Omega^i \right)' \Lambda_{t-1}^i \right], \tag{14}$$

with

$$\Omega^i \equiv \begin{bmatrix} I & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \Omega_x^i & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \Omega_v^i \end{bmatrix},$$

and where ϖ^i is defined as the asymptotic speed of adjustment in the learning algorithm, as described in Honkapohja and Mitra (2006). Since we assume agents in each country apply the same type of learning algorithm, we have $\varpi^H = \varpi^F = \varpi$. Further, denote any possible RPE that satisfies the conditions (11)-(13) by $\left(\hat{\Lambda}^H, \hat{\Lambda}^F\right)$. Then, by linearizing around the RPE and vectorizing the right-hand side of the differential equations (14), one obtains

$$\begin{bmatrix} \frac{d\text{vec}\left(\Lambda^H\right)}{d\tau} \\ \frac{d\text{vec}\left(\Lambda^F\right)}{d\tau} \end{bmatrix} = \varpi \Xi \begin{bmatrix} \text{vec}\left(\Lambda^H\right) \\ \text{vec}\left(\Lambda^F\right) \end{bmatrix}, \tag{15}$$

where τ expresses 'notional time' as in Evans and Honkapohja (2001) and Ξ is given by

$$\Xi = \begin{bmatrix} \left(\Omega_x^H \otimes I\right) D_{\Lambda^H} T - I \otimes \left(\Omega^H\right)' & \left(\Omega_x^H \otimes I\right) D_{\Lambda^F} T \\ \left(\Omega_x^F \otimes I\right) D_{\Lambda^H} T & \left(\Omega_x^F \otimes I\right) D_{\Lambda^F} T - I \otimes \left(\Omega^F\right)' \end{bmatrix}, \tag{16}$$

with

$$D_{\Lambda^i} T \left(\hat{\Lambda}^H, \hat{\Lambda}^F\right) = \begin{bmatrix} \Omega_x^i F^i \left(I + \hat{\Lambda}_1^i \Omega_x^i\right) & \left(\Omega_x^i \hat{\Lambda}_0^i\right)' \otimes \Omega_x^i F^i & \mathbf{0} \\ \mathbf{0} & \begin{matrix} (\Omega_x^i)' \otimes \Omega_x^i F^i \hat{\Lambda}_1^i \Omega_x^i + \\ \left(\Omega_x^i \hat{\Lambda}_1^i \Omega_x^i\right)' \otimes \Omega_x^i F^i \end{matrix} & \mathbf{0} \\ \mathbf{0} & \left(\Omega_x^i \hat{\Lambda}_2^i \Omega_v^i\right)' \otimes \Omega_x^i F^i & \begin{matrix} (\Omega_v^i)' \otimes \Omega_x^i F^i \hat{\Lambda}_1^i \Omega_x^i + \\ (\Omega_v^i)' \varrho' \otimes \Omega_x^i F^i \end{matrix} \end{bmatrix}. \tag{17}$$

It follows that, under home biased expectations, E-stability of the RPE requires all eigenvalues of Ξ have negative real parts (Evans and Honkapohja, 2001).

We are now ready to present the requirements for E-stability of the model under home biased expectations:

Proposition 1. *The RPE of the model with home bias in expectations is E-stable, if and only if the eigenvalues of* Ξ *have negative real parts.*

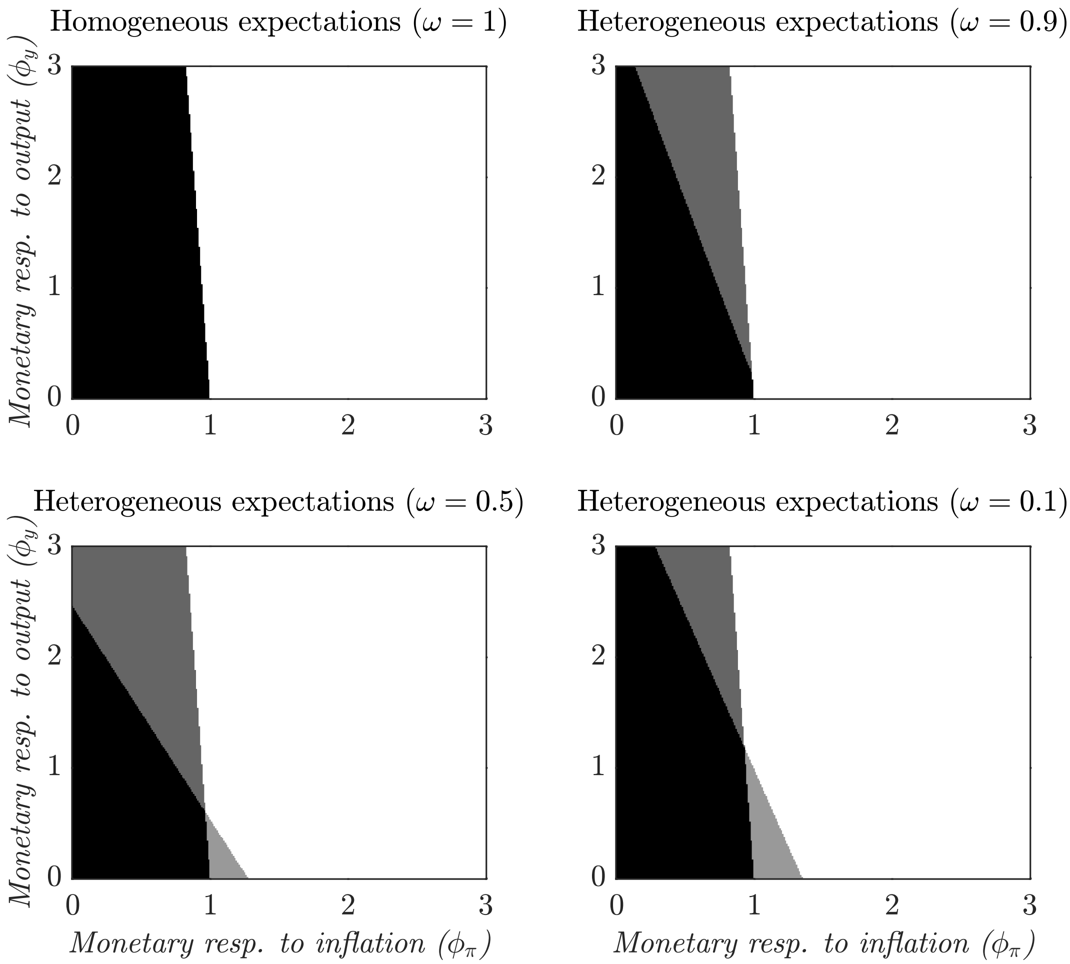
Next, we investigate the ability of monetary policy to deliver E-stability of the RPE under different degrees of home bias in expectations.

5.2 Monetary requirements for (E-)stability

Figure 6 shows the conditions for E-stability from Proposition 1 as a function of the monetary policy parameters $\phi_\pi \in [0, 3]$, which governs the monetary response to inflation, and $\phi_y \in [0, 3]$, which governs the monetary response to output. The remaining model parameters are kept at their benchmark values, with the exception of α which we now set to 0.6 to ensure existence of an RPE over the whole grid of policy parameters. In the white area, monetary policy yields stable and unique rational expectations equilibria. These equilibria are also E-stable, i.e. learnable, RPEs. In the dark gray area, equilibrium is indeterminate, yet the RPE is E-stable. In the light gray area, the REE is determinate, whereas the RPE is *not* E-stable. Finally, the black region indicates the parameter space in which monetary policy is unable to deliver a determinate REE nor an E-stable RPE.

In the upper-left panel of Figure 6, expectations are free from home bias. In this case, we find that the Taylor-principle is a sufficient condition for both determinacy and E-stability. In other words, the central bank must respond to increases (decreases) in inflation by raising (lowering) the nominal interest rate by more than one for one, which requires $\phi_\pi > 1$. When the monetary policy response to output is sufficiently strong, ϕ_π can even be set slightly below unity.

Figure 6: Determinacy and E-stability for different degrees of home bias in expectations



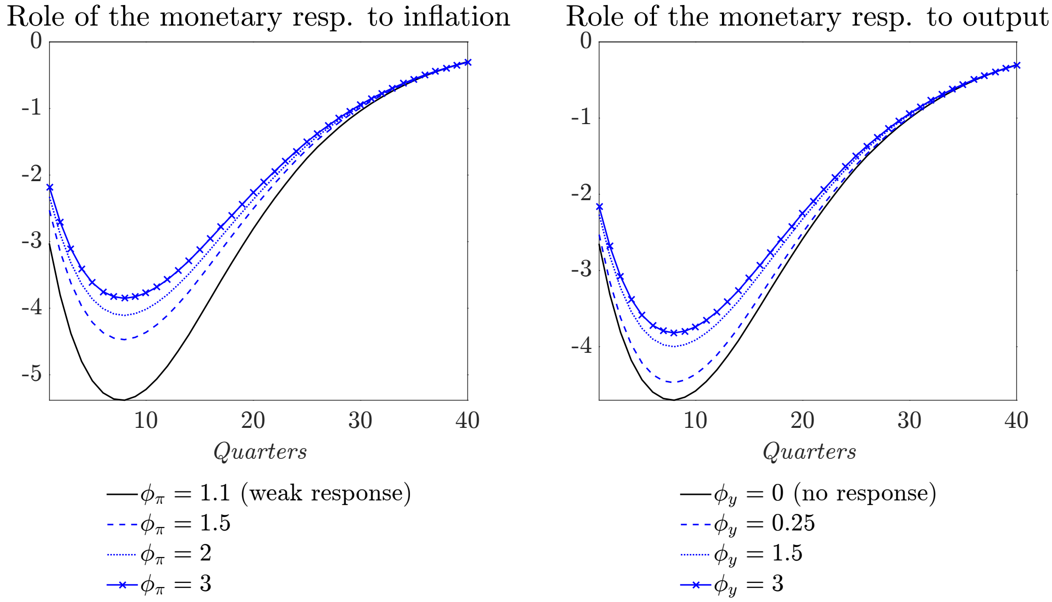
Note: The figure shows the conditions for E-stability and determinacy as a function of the monetary policy parameters ϕ_π and ϕ_y, and conditional on different degrees of home bias in expectations. White = determinate Rational Expectations Equilibrium (REE) and E-stable Restricted Perception Equilibrium (RPE); dark gray = indeterminate REE and E-stable RPE; light gray = determinate REE and E-unstable RPE; black = indeterminate REE and E-unstable RPE.

The remaining panels of Figure 6 show that the determinacy and E-stability requirements for monetary policy change when expectations are home biased. In fact, the monetary union is more likely to fall in regions characterized by E-unstable RPEs, the greater is the degree of home bias in expectations, which is reflected by an expansion of the light-gray region as the home bias in expectations increases (i.e. as ω falls). To guarantee an equilibrium that is both determinate and E-stable under home biased expectations, the central bank must respond more aggressively to changes in inflation or output (or both). Thus, the monetary policy trade-off between inflation and output stabilization worsens under home biased expectations.

These results are closely related to the Walters critique (Walters, 1994). Alan Walters, in his plea against UK membership of the European Monetary System, argued that monetary unions are inherently unstable due to the inability of member countries to, unilaterally, offset country-specific shocks through an adjustment in the nominal interest rate or nominal exchange rate. Accordingly, countries belonging to a monetary union would be more susceptible to self-strengthening inflationary or deflationary cycles than countries operating under flexible exchange rates. Those who challenge the Walters critique point to the stabilizing effects of the real exchange rate channel that, at least in the long run, dominate the pro-cyclical effects of the real interest rate channel. The impulse responses shown in Section 4 indicate that both channels are relevant for macroeconomic stability and that the relative strengths of the two channels depend on the degree of home bias in expectations.

The results from the E-stability analysis suggest that monetary unions may indeed be more likely to be unstable, thereby supporting the Walters critique, yet only insofar expectations are home biased. If the home bias in expectations does not exist, we find that the stability and determinacy requirements for monetary policy are not that different from those pertaining to economies outside monetary union (Clarida et al., 1999). Yet, if expectations are subject to home bias, stability of the union is threatened. In that case, to avoid unstable macroeconomic outcomes, the central bank must adopt a more aggressive stance towards inflation and output.

Figure 7: Macroeconomic imbalances and monetary policy



Note: The figure shows the response of Home's trade balance to an asymmetric demand shock under home biased expectations, with $\omega = 0.1$. Although we used a learning algorithm with decreasing gain, the results are nearly identical under a constant gain of $\gamma = 0.01$, which was used in Section 4.

To see whether a more aggressive monetary policy stance helps mitigate macroeconomic imbalances under home biased expectations, we again examine the effects of an asymmetric demand shock on Home's trade balance for $\omega = 0.1$ and different values of the monetary policy parameters ϕ_π and ϕ_y. The results are shown in Figure 7.

According to the figure, a more aggressive response of the central bank to changes in either inflation (i.e. higher values for ϕ_π, see left panel) or output (higher values for ϕ_y, right panel) reduces the trade balance's response to the asymmetric demand shock. Recall that, following the shock, the nominal interest rate is left unchanged as union-wide aggregates remain constant. However, different monetary policy stances still can deliver different macroeconomic dynamics, even without changing the nominal interest rate, due to their effect on agents' expectations *before* the shock hits the economy. For instance, following an increase in aggregate demand, agents' inflation expectations will remain closer to the inflation target if they experienced a central bank that, in the past, was more hawkish rather than dovish. Therefore, a more aggressive monetary policy stance helps weaken the feedback between inflation expectations and the real interest rate, thereby also weakening the real interest rate channel and allowing the real exchange rate to stabilize economic conditions more promptly.

Although the benefits of having anchored inflation expectations, in terms of promoting macroeconomic stability, are well understood, the results shown here provide an additional argument in favor of tightly anchored inflation expectations that applies particularly to monetary unions: not only does a credible inflation target prevent expectations from drifting too far from steady state, it also helps prevent expectations from *diverging* too far across regions within the union following country-specific shocks.

6 Conclusion

Although the importance of expectations for policy design and evaluation is broadly acknowledged in the literature, attention is often focused on (rational) expectations at the aggregate level, thereby ignoring potentially important differences in expectations across agents. This approach does not seem appropriate when studying asymmetric shocks under monetary union, where forces that provoke divergences in economic dynamics and expectations across agents from different member countries are likely to be strong.

In this paper, we studied the role of home bias in expectations for macroeconomic imbalances in a monetary union. We provided empirical evidence that suggests expectations can become more biased towards domestic variables in times of large country-specific shocks. We then applied a New Keynesian model for a two-country monetary union that features such home bias in expectations and examined the implications for the size and duration of macroeconomic imbalances following asymmetric demand shocks. We found that home bias in expectations aggravates pro-cyclical real interest rate effects, causing macroeconomic imbalances be more pronounced. In particular, the expectations bias shifts agents' attention from international price competitiveness effects to domestic real interest rate effects. As a result, overheating economic conditions in countries with above-average inflation tend to persist, since the relatively low real interest rate keeps economic activity elevated despite the appreciation of the real exchange rate. In contrast, countries with below-average inflation and high real interest rates experience more protracted recessions. In both cases, the real exchange rate must adjust more strongly in order to normalize economic conditions than if expectations were rational or free from home bias. We further showed that, to prevent expectations from drifting too far from the currency area's inflation target, monetary policy must target inflation and output more aggressively, especially if the degree of home bias in expectations is large.

It is well established that cross-country inflation differentials under monetary union may reflect, at least in part, the endogenous response of the real exchange rate to asymmetric shocks. In fact, such a self-correcting mechanism forms an integral part of the smooth functioning of monetary unions (Angeloni and Ehrmann, 2007). Hence, it is not always clear whether a central bank should care about dispersion in inflation rates across member countries. However, the results presented in this paper point towards a potential risk embedded in divergent inflationary dynamics under monetary union that is related to the way agents form expectations. To distinguish between regional economic disparities that are either temporary, and part of a necessary adjustment process, or structural, and potentially unstable, the central bank should monitor expectations at both the aggregate and national level. Internalizing interregional dispersion into the monetary policy strategy, then, is warranted insofar expectations are highly sensitive to past domestic economic conditions and ignorant of international spillover effects. Alternatively, fiscal policy could be employed to offset the bias in expectations at the national level, for instance by changing import tariffs or export subsidies to replicate the required real exchange rate adjustment. Whether such policies are able to fully undo the effects of the home bias in expectations is a question we leave for future research.

References

Allsopp, C. and Vines, D. (2010). *The euro: The first decade*, chapter Fiscal policy, intercountry adjustment and the real exchange rate, pages 511–551. Cambridge University Press.

Angeloni, I. and Ehrmann, M. (2007). Euro area inflation differentials. *The BE Journal of Macroeconomics*, 7(1).

Autrup, S. L. and Grothe, M. (2014). Economic surprises and inflation expectations: Has anchoring of expectations survived the crisis? ECB working paper no 1671, European Central Bank.

Beechey, M. J., Johannsen, B. K., and Levin, A. T. (2011). Are long-run inflation expectations anchored more firmly in the euro area than in the United States? *American Economic Journal: Macroeconomics*, 3(2):104–129.

Berardi, M. (2007). Heterogeneity and misspecifications in learning. *Journal of Economic Dynamics and Control*, 31(10):3203–3227.

Berardi, M. (2009). Monetary policy with heterogeneous and misspecified expectations. *Journal of Money, Credit and Banking*, 41(1):79–100.

Berardi, M. and Duffy, J. (2007). The value of central bank transparency when agents are learning. *European Journal of Political Economy*, 23(1):9–29.

Bernanke, B. S. (2007). Inflation expectations and inflation forecasting. In *Speech at the Monetary Economics Workshop of the National Bureau of Economic Research Summer Institute, Cambridge, Massachusetts.*

Branch, W. A. and Evans, G. W. (2006a). Intrinsic heterogeneity in expectation formation. *Journal of Economic Theory*, 127(1):264–295.

Branch, W. A. and Evans, G. W. (2006b). A simple recursive forecasting model. *Economics Letters*, 91(2):158–166.

Branch, W. A. and Evans, G. W. (2011). Monetary policy and heterogeneous expectations. *Economic Theory*, 47(2):365–393.

Branch, W. A., Evans, G. W., and McGough, B. (2014). Perpetual learning and stability in macroeconomic models. Unpublished manuscript.

Branch, W. A. and McGough, B. (2009). A New Keynesian model with heterogeneous expectations. *Journal of Economic Dynamics and Control*, 33(5):1036–1051.

Bullard, J. and Mitra, K. (2002). Learning about monetary policy rules. *Journal of Monetary Economics*, 49(6):1105–1129.

Calvo, G. A. (1983). Staggered prices in a utility-maximizing framework. *Journal of Monetary Economics*, 12(3):383–398.

Carlin, W. (2013). Real exchange rate adjustment, wage-setting institutions, and fiscal stabilization policy: Lessons of the eurozone's first decade. *CESifo Economic Studies*, 59(3):489–519.

Carroll, C. D. (2003). Macroeconomic expectations of households and professional forecasters. *The Quarterly Journal of economics*, 118(1):269–298.

Clarida, R., Gali, J., and Gertler, M. (1999). The science of monetary policy: A New Keynesian perspective. *Journal of Economic Literature*, 37(4):1661–1707.

Coibion, O., Gorodnichenko, Y., and Kumar, S. (2015). How do firms form their expectations? New survey evidence. NBER Working Paper 21092, National Bureau of Economic Research.

Deroose, S., Roeger, W., and Langedijk, S. (2008). Reviewing adjustment dynamics in EMU: From overheating to overcooling. Economic Papers 198, European Economy, European Commission.

Dias, F., Duarte, C., and Rua, A. (2010). Inflation (mis) perceptions in the euro area. *Empirical economics*, 39(2):353–369.

ECB (2012). Assessing the anchoring of longer-term inflation expectations. *Monthly Bulletin*, July:65–78.

Ehrmann, M., Fratzscher, M., Gürkaynak, R. S., and Swanson, E. T. (2011). Convergence and anchoring of yield curves in the euro area. *The Review of Economics and Statistics*, 93(1):350–364.

Eusepi, S. and Preston, B. (2011). Expectations, learning, and business cycle fluctuations. *The American Economic Review*, 101(6):2844–2872.

Evans, G. W. and Honkapohja, S. (2001). *Learning and expectations in macroeconomics*. Princeton University Press.

Evans, G. W. and Honkapohja, S. (2003). Expectations and the stability problem for optimal monetary policies. *The Review of Economic Studies*, 70(4):807–824.

Evans, G. W. and McGough, B. (2007). Optimal constrained interest-rate rules. *Journal of Money, Credit and Banking*, 39(6):1335–1356.

Gabaix, X. (2014). A sparsity-based model of bounded rationality. *The Quarterly Journal of Economics*, 129(4):1661–1710.

Galati, G., Poelhekke, S., and Zhou, C. (2011). Did the crisis affect inflation expectations? *International Journal of Central Banking*, 7:167–207.

Gasteiger, E. (2014). Heterogeneous expectations, optimal monetary policy, and the merit of policy inertia. *Journal of Money, Credit and Banking*, 46(7):1535–1554.

Honkapohja, S. and Mitra, K. (2006). Learning stability in economies with heterogeneous agents. *Review of Economic Dynamics*, 9(2):284–309.

Madeira, C. and Zafar, B. (2015). Heterogeneous inflation expectations and learning. *Journal of Money, Credit and Banking*, 47(5):867–896.

Mankiw, N. G. and Reis, R. (2002). Sticky information versus sticky prices: A proposal to replace the New Keynesian Phillips curve. *Quarterly Journal of Economics*, 117(4):1295–1328.

Massaro, D. (2013). Heterogeneous expectations in monetary DSGE models. *Journal of Economic Dynamics and Control*, 37(3):680–692.

Mishkin, F. S. (2007). Inflation dynamics. *International Finance*, 10(3):317–334.

Orphanides, A. and Williams, J. C. (2005). Inflation scares and forecast-based monetary policy. *Review of Economic Dynamics*, 8(2):498–527.

Pfajfar, D. and Santoro, E. (2010). Heterogeneity, learning and information stickiness in inflation expectations. *Journal of Economic Behavior & Organization*, 75(3):426–444.

Primiceri, G. E. (2005). Time-varying structural vector autoregressions and monetary policy. *The Review of Economic Studies*, 72(3):821–852.

Santoro, S. (2017). Heterogeneity and learning with complete markets. *Economic Theory*, 64(1):183–211.

Simon, H. A. (1984). On the behavioral and rational foundations of economic dynamics. *Journal of Economic Behavior & Organization*, 5(1):35–55.

Sims, C. A. (2003). Implications of rational inattention. *Journal of Monetary Economics*, 50(3):665–690.

Toroj, A. (2010). Rationality of expectations: Another OCA criterion? A DSGE analysis. *Central European Journal of Economic Modelling and Econometrics*, 3(2):205–252.

Van Nieuwerburgh, S. and Veldkamp, L. (2009). Information immobility and the home bias puzzle. *The Journal of Finance*, 64(3):1187–1215.

Walters, A. (1994). Walters critique. *The Economics and Politics of Money: The Selected Essays of Alan Walters*, pages 368–370. Edward Elgar Publishing, Northampton.

A The linear model

The full non-linear model is presented in the Online Appendix. Here, we present the linear version of the model. The model is linearized around a deterministic, zero-inflation steady state. Let variables with a hat denote the percentage deviation of the corresponding variable from its steady-state value, i.e. $\hat{x}_t = (x_t - x)/x$ for any generic variable x_t. A scaled-down version of the linear model then consists of the following equations.

First, the Euler equation for the Home country and the imperfect international risk sharing condition:[15]

$$\sigma \hat{c}_t^H = \sigma \tilde{E}_t^H \hat{c}_{t+1}^H - \left(\hat{R}_t - \tilde{E}_t^H \hat{\pi}_{t+1}^H \right) + (1 - \rho_D) z_{D,t}^H, \tag{18}$$

$$\sigma \left(\hat{c}_t^F - \hat{c}_t^H \right) = \sigma \left(\tilde{E}_t^F \hat{c}_{t+1}^F - \tilde{E}_t^H \hat{c}_{t+1}^H \right) + \tilde{E}_t^F \hat{\pi}_{t+1}^F - \tilde{E}_t^H \hat{\pi}_{t+1}^H + (1 - \rho_D) \left(z_{D,t}^F - z_{D,t}^H \right), \tag{19}$$

where $\hat{c}_t^i$ denotes consumption in country $i = \{H, F\}$, $\hat{R}_t$ the nominal interest rate set by the central bank, and $\hat{\pi}_t^i$ CPI inflation, and where $\sigma > 0$ measures the inverse of the elasticity of intertemporal substitution. $z_{D,t}^i$ is a demand shock that evolves according to

$$z_{D,t}^i = \rho_D z_{D,t-1}^i + \varepsilon_{D,t}^i, \tag{20}$$

with $\rho_D \in [0, 1]$ and $\varepsilon_{D,t}^i \sim \mathcal{N}(0, \sigma^2)$.

Second, two resource constraints:

$$\hat{y}_t^H = \left(1 - \overline{\alpha}^H \right) \frac{c^H}{y^H} \hat{c}_t^H + \frac{1-s}{s} \overline{\alpha}^F \frac{c^F}{y^H} \hat{c}_t^F + \Theta_H \hat{q}_t, \tag{21}$$

$$\hat{y}_t^F = \left(1 - \overline{\alpha}^F \right) \frac{c^F}{y^F} \hat{c}_t^F + \frac{s}{1-s} \overline{\alpha}^H \frac{c^H}{y^F} \hat{c}_t^H - \Theta_F \hat{q}_t, \tag{22}$$

with $\hat{y}_t^i$ domestic output, $\hat{q}_t$ Home's real exchange rate, and where $\hat{\alpha}^i = \overline{\alpha}^i / \left(1 - \overline{\alpha}^F - \overline{\alpha}^H \right)$,

[15]The imperfect international risk sharing condition is derived in the Online Appendix.

with $\overline{\alpha}^H \equiv (1-s)\,\alpha$, $\overline{\alpha}^F \equiv s\alpha$ and

$$\Theta_H \equiv \eta \frac{\hat{\alpha}^H}{\overline{\alpha}^H} \left[\overline{\alpha}^H \left(1-\overline{\alpha}^H\right) \frac{c^H}{y^H} + \left(1-\overline{\alpha}^F\right) \frac{1-s}{s} \overline{\alpha}^F \frac{c^F}{y^H} \right],$$
$$\Theta_F \equiv \eta \frac{\hat{\alpha}^F}{\overline{\alpha}^F} \left[\overline{\alpha}^F \left(1-\overline{\alpha}^F\right) \frac{c^F}{y^F} + \left(1-\overline{\alpha}^H\right) \frac{s}{1-s} \overline{\alpha}^H \frac{c^H}{y^F} \right].$$

The parameter $s \in [0,1]$ measures the size of Home relative to Foreign, while $\eta \geq 1$ governs the trade elasticity and $\alpha \in [0,1]$ the import-share in household consumption.

Third, two conditions for CPI inflation and one that governs the dynamics of Home's real exchange rate:

$$\hat{\pi}_t^H = \hat{\pi}_{H,t}^H + \hat{\alpha}^H \left(\hat{q}_t - \hat{q}_{t-1}\right), \tag{23}$$
$$\hat{\pi}_t^F = \hat{\pi}_{F,t}^F - \hat{\alpha}^F \left(\hat{q}_t - \hat{q}_{t-1}\right), \tag{24}$$
$$\hat{q}_t = \hat{q}_{t-1} + \hat{\pi}_t^F - \hat{\pi}_t^H, \tag{25}$$

with $\hat{\pi}_{i,t}^i$ PPI inflation.

Fourth, two New Keynesian Phillips curves:

$$\hat{\pi}_{H,t}^H = \beta \tilde{E}_t^H \hat{\pi}_{H,t+1}^H + \kappa \left(\varphi \hat{y}_t^H + \sigma \hat{c}_t^H + \hat{\alpha}^H \hat{q}_t\right), \tag{26}$$
$$\hat{\pi}_{F,t}^F = \beta \tilde{E}_t^F \hat{\pi}_{F,t+1}^F + \kappa \left(\varphi \hat{y}_t^F + \sigma \hat{c}_t^F - \hat{\alpha}^F \hat{q}_t\right), \tag{27}$$

with $\beta \in (0,1)$ the household's discount factor, $\varphi > 0$ the inverse of the Frisch elasticity of labor supply, and where $\kappa \equiv (1-\theta)(1-\beta\theta)/\theta$ with $\theta \in (0,1)$ the probability of non-price adjustment.

And finally, the monetary policy rule of the single central bank:

$$\hat{R}_t = \phi_\pi \left[s\hat{\pi}_t^H + (1-s)\,\hat{\pi}_t^F\right] + \phi_y \left[s\hat{y}_t^H + (1-s)\,\hat{y}_t^F\right], \tag{28}$$

with $\phi_\pi > 1$ and $\phi_y \geq 0$.

Table 2: Benchmark calibration

β	Household discount factor	0.99
$1/\varphi$	Frisch elasticity of labor supply	1
$1/\sigma$	Elasticity of intertemporal substitution	1
α	Import-share of consumption	0.25
θ	Probability of non-price adjustment	0.75
η	Trade elasticity	1
ϕ_π	Monetary response to inflation	1.5
ϕ_y	Monetary response to output	0.25
s	Relative size of Home	0.5
ρ_D	AR(1) coefficient of the demand shock	0.9

For $x_t \equiv \left[x_t^H,\ x_t^F,\ \hat{q}_t,\ \hat{R}_t\right]'$, where $x_t^i \equiv \left[\hat{y}_t^i,\ \hat{c}_t^i,\ \hat{\pi}_t^i,\ \hat{\pi}_{i,t}^i\right]'$, and $v_t \equiv \left[z_{D,t}^H,\ z_{D,t}^F\right]'$, the state-space representation of the model is given by Equations (3) and (4) in Section 3.1.

Table 2 presents the benchmark calibration of the model parameters.

B Derivation of Equation (14)

To derive the ordinary differential equation (14), first consider (7) and substitute in for $\tilde{x}_t^i$ by multiplying (9) by Ω_x^i to find

$$\begin{aligned}
\Lambda_t^i &= \Lambda_{t-1}^i + \gamma_t \left(M_t^i\right)^{-1} \tilde{z}_t^i \left[\Omega_x^i T\left(\Lambda_{t-1}^H, \Lambda_{t-1}^F\right) z_t - \Lambda_{t-1}^i \tilde{z}_t^i\right]' \\
&= \Lambda_{t-1}^i + \gamma_t \left(M_t^i\right)^{-1} \Omega^i z_t \left[\Omega_x^i T\left(\Lambda_{t-1}^H, \Lambda_{t-1}^F\right) z_t - \Lambda_{t-1}^i \Omega^i z_t\right]' \\
&= \Lambda_{t-1}^i + \gamma_t \left(M_t^i\right)^{-1} \Omega^i z_t \left\{\left[\Omega_x^i T\left(\Lambda_{t-1}^H, \Lambda_{t-1}^F\right) - \Lambda_{t-1}^i \Omega^i\right] z_t\right\}' \\
&= \Lambda_{t-1}^i + \gamma_t \left(M_t^i\right)^{-1} \Omega^i z_t z_t' \left[\Omega_x^i T\left(\Lambda_{t-1}^H, \Lambda_{t-1}^F\right) - \Lambda_{t-1}^i \Omega^i\right]'.
\end{aligned}$$

Note that

$$E\left[\left(M_t^i\right)^{-1} \Omega^i z_t z_t' \begin{pmatrix} \left\{\Omega_x^i \left[\sum_{j=H,F} F^j \left(I + \Lambda_1^j \Omega_x^j\right) \Lambda_0^j\right] - \Lambda_0^i\right\}' \\ \left\{\Omega_x^i \left[\sum_{j=H,F} F^j \left(\Lambda_1^j \Omega_x^j\right)^2 + G\right] - \Omega_x^i \Lambda_1^i\right\}' \\ \left\{\Omega_x^i \left[\sum_{j=H,F} F^j \left(\Lambda_1^j \Omega_x^j \Lambda_2^j + \Lambda_2^j \varrho\right) \Omega_v^j + H\right] - \Omega_v^i \Lambda_2^i\right\}' \end{pmatrix}\right].$$

Let $E[zz'] \equiv \lim_t E[z_t z_t']$. Then, we obtain the following ODEs:

$$\frac{d\Lambda_i}{d\tau} = \varpi^i \left(M_t^i\right)^{-1} \Omega^i E[zz'] \left[\Omega_x^i T\left(\Lambda_{t-1}^H, \Lambda_{t-1}^F\right) - \Lambda_{t-1}^i \Omega^i\right]', \tag{29}$$

$$\frac{dM^i}{d\tau} = \varpi^i \left[\Omega^i E[zz'] - M^i\right]. \tag{30}$$

From Equation (30), it is evident that $M^i \to \Omega^i E[zz']$, which implies that Equation (29) reduces to Equation (14).