Symbiosis and Coordination of Macroeconomic Policies in a Monetary Union

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Abstract

This paper deals with strategic policy interactions in a monetary union. We use a static two-country monetary-union model, which incorporates the key features of the New-Keynesian framework. We investigate the policy mix outcome under non-conflicting but different objectives when the two policy instruments can directly affect inflation. Thus, we provide a reconciliation of the early literature, which is mostly based on the supply-side of the economy with the most recent literature, which mainly focuses on the demand side. We consider the short-run macroeconomic stabilization and welfare implications of the fiscal-monetary policy interactions at both the union and national levels. We compare and contrast the alternative strategic regimes (simultaneous-move, fiscal/monetary leadership) in the monetary union and we analyze both the horizontal (across governments) and the vertical (between the monetary and the fiscal authorities) coordination problems. We define the impact that the policies’ direct effects on inflation has on (i) fiscal authorities’ cooperation, (ii) policies’ cyclicality, and (iii) the alternative strategic regimes (symbiosis). We draw important results on the preferable strategic and fiscal regimes for the monetary authority.
Keywords: Monetary union; Fiscal/monetary policies; Coordination; Symbiosis; Strategic regimes.

JEL Classification: E52; E61; E62; E63; F45.

1. Introduction

There has been more than fifteen years since the official launch of the Economic and Monetary Union (EMU) in Europe, the “greatest monetary reform since Bretton Woods” (Buti, 2003, p. 24). However, the consequences of fiscal and monetary policy interactions still remain an issue among both academia and policymakers. The recent travails of the Eurozone reveal that the institutional structure of policymaking has been imperfect and motivate further research on fiscal-monetary policy interactions in monetary unions. Monetary policy is conducted by an independent supranational authority, the European Central Bank (ECB), while fiscal policy remains decentralized at national level, respecting the debt sustainability constraint imposed by the European Union (EU), meaning the Stability and Growth Pact (SGP), and more recent fiscal developments described by the Fiscal Compact\(^2\) (FC). In this framework, fiscal policy remains the macroeconomic tool of national authorities to stabilize their economies under country-specific shocks. However, non-coordinated fiscal policies in national level may create externalities to other member-states, creating inefficiencies. This might induce the possibility of policy coordination.

Beetsma and Giuliodori (2010) provide an overview of the research on the macroeconomic costs and benefits of the EMU. In Sections 6 and 7, the authors examine fiscal policy and conflicts of interest in the monetary union, as well as fiscal spillovers and coordination. Formal analysis of the policy mix and fiscal policies’ coordination requires a framework to model strategic interactions among fiscal authorities and the common central bank.\(^3\) In particular, assumptions regarding authorities’ objectives, their ability to commit and the timing of their decisions are at the center of the analysis. In a recent paper, Foresti (2017) analyzes the literature on strategic fiscal/monetary policy interactions in a monetary union. The author presents a generic theoretical framework in order to highlight the main points of the literature, regarding uncertainty issues, authorities’ preferences, the role of commitment to policy rules, and coordination. All these issues have regained interest due to the Eurozone sovereign debt crisis, being part of the appropriate institutional framework of policymaking in the EMU.

Following Plasmans et. al. (2006), the literature has been mainly focused on two policy interactions: (i) the links between deficits, debts, inflation and interest rates via the (dynamic) government budget constraints, and (ii) the links between fiscal and monetary policies in a macroeconomic stabilization

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\(^2\) Its official name is “The Treaty on Stability, Coordination and Governance” (TSCG).

\(^3\) To quote Fragetta and Kirsanova (2010, p. 856), “…there is little doubt that authorities can act strategically”.
perspective. This paper follows the second strand of the literature, abstracting from important long-run issues that are related to fiscal policy, such as debt sustainability. To quote Uhlig (2003, p. 43), we are dealing with the ‘...day-to-day policy task of responding to business cycle shocks’. The literature so far has offered a plethora of different modeling assumptions that provided mixed results, while general conditions for cooperation and commitment irrelevance provided by Kempf and von Thadden (2013) follow the lines of the traditional Barro-Gordon (1983) set-up, where the effects of monetary and fiscal policies are often set to work only on the supply side of the economy. According to Plasman et al. (2006), such an approach seems rather narrow, considering also that the supply-side effects of monetary and fiscal policies may in practice be of limited relevance, as they often take a very long time to materialize. On the contrary, the most recent literature is based on the New-Keynesian framework, which focuses on the demand side of the economy, where supply is often held fixed.

This paper proposes a unified theoretical framework to analyze strategic policy interactions in a monetary union. We use a static representation of the New-Keynesian model, following mainly Andersen (2005, 2008). We assume that the two (for simplicity) member-states in the monetary union are interconnected via a trade effect and a terms-of-trade effect, and that the policy instruments, namely the country-specific fiscal stances and the common nominal interest rate, can also directly affect country-specific inflation. Fiscal policy can have either positive or negative direct effects on inflation, as various fiscal instruments can have (positive/negative) short-run effects on the supply-side of the economy (see, e.g., Andersen, 2005, 2008; Debrun, 2000), whereas monetary policy can have a direct positive effect on inflation, following mainly the cost channel (Ravenna and Walsh, 2006). Our motivation is to provide a reconciliation between the early literature that was mainly based on the supply-side of the economy with the most recent one that is mainly focused on the demand side. In the latter case, the Phillips curve is only affected by the output gap, which means that the two policy instruments are perfect substitutes in the stabilization process. By comparing the two cases, we mainly focus on the ordering of moves and the resulting cyclical behavior on the part of the authorities, for both cases of decentralized and centralized fiscal policies, where the latter case defines fiscal authorities’ cooperation. We thus investigate the policy mix and the coordination problem in a monetary union under non-conflicting but different objectives, when both policy instruments can directly affect inflation. Beetsma and Debrun (2004) distinguish the coordination problem between a horizontal (across governments) coordination problem and a vertical (between the monetary and the fiscal authorities) one. By non-conflicting objectives we mean that all the authorities agree on the ideal targets of the concerned macroeconomic variables, being their long-run

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4 For example, Aguiar et al. (2015) study fiscal and monetary policies in a monetary union with the potential for rollover crises in sovereign debt markets.
equilibrium values (Uhlig, 2003). However, objectives may differ, as the national fiscal authorities care about fiscal stance stabilization and not about inflation. This creates the policy conflict (Kempf and von Thadden, 2013).

In a series of critical papers, Dixit and Lambertini (2001, 2003a) studied strategic policy interactions (pure macroeconomic stabilization) in a monetary union in a Barro-Gordon (1983) framework where fiscal policy can also affect (common) inflation, together with monetary policy, while the monetary authority is concerned with country-specific data. The main results are: (i) under conflicting objectives, the simultaneous-move strategic regime is inferior to any leadership regime, while (ii) under non-conflicting objectives, there is symbiosis of monetary and fiscal policies, in that the actual targets can be obtained irrespective of the ordering of moves, of fiscal authorities’ cooperation or of identical preference priorities. Under conflicting objectives, the Nash game produces a sub-optimal race with fiscal expansion aimed at raising output and monetary contraction aimed at offsetting the effect of fiscal expansion on inflation, which yields extreme outcomes. The leadership regime, instead, produces improved outcomes, as the leader moderates its policy in anticipation of the follower’s reaction, who moderates its policy, too.5 Beetsma and Bovenberg (1998), following Alesina and Tabellini (1987), assume that the monetary authority directly controls the common inflation rate, while they also incorporate a government budget constraint. The authors find that fiscal policies’ coordination is welfare-reducing, as it makes the fiscal authorities to set a high tax rate in order to induce a relax of their budget constraints through an expansionary monetary policy, hence strengthening their strategic position relative to the monetary authority. In this model, monetary unification is welfare-enhancing.

Kempf and von Thadden (2013) provide the general conditions for the irrelevance of the ECB’s commitment capacity and the sequencing of moves (symbiosis result), as well as for fiscal policies’ coordination irrelevance, in monetary unions under both private and fiscal spillovers, combining the work of Dixit and Lambertini (2001, 2003a) and Chari and Kehoe (2008) in a unified framework. The private spillovers refer to the (wage) decisions by (multiple) private agents (non-coordinated wage setters) within countries (Chari and Kehoe, 2008).6 The monetary authority is concerned with country-specific data, there is a common inflation rate, and the comparison between the two fiscal regimes is made on union-wide equilibrium solutions. The authors consider alternative commitment patterns (leadership regimes) in that each player (the private sector; fiscal authorities; the monetary authority) moves at a particular stage of the game, where all private agents act at the same stage. The fiscal authorities act at the same stage, too. The

5 In a closed-economy setting, the superiority of the fiscal leadership regime is also stressed by Dixit and Lambertini (2003b) and Hughes Hallett and Weymark (2007), as it provides a regime of implicit coordination between the authorities.

6 In the Barro-Gordon (1983) framework, instead, there exists a representative private sector.
sufficient conditions are that the direct spill-over effects must have no strategic significance and that the number of instruments must match the number of squared gaps in all authorities’ payoff functions. If further all the authorities agree on the objectives, then the bliss points can be also achieved. In the absence of those conditions, both cooperation and commitment (the sequence of moves) matter, while the difference between the non-cooperative and the cooperative outcome depends on the number of countries in the monetary union. The authors clearly show that the monetary union benefits from fiscal authorities’ cooperation under fiscal leadership.

The above discussion on the symbiosis result shows that it only holds under specific assumptions of the model. In the opposite case, both coordination and timing issues become relevant, creating a policy-mix bias (Foresti, 2017). Since the official launch of the EMU, there are a lot of papers that deal with the macroeconomic policy mix, fiscal authorities’ cooperation, and the sequencing of moves in monetary unions. Banerjee (2001) allows for fiscal policies to be subject to potential time inconsistencies, while comparing different scenarios of commitment and discretion. The author finds that moving from the scenario of full discretion to other scenarios results in lower inflation and higher expenditure at the expense of lower output for the full commitment and the fiscal commitment ones, whereas for the monetary commitment we end up with lower public spending. Godbillon and Sidiropoulos (2001) show that delegation of fiscal policy to a council of country representatives and monetary policy to a council of governors is the appropriate institutional design to reduce the inflation bias and better stabilize regional idiosyncratic supply and demand shocks in a monetary union. Lambertini and Rovelli (2004) show that a ‘vertical’ coordination problem arises even in an extremely simple setting of a simultaneous-move game in a static two-country monetary-union model, where the two countries are identical and there are no interconnections between them. The authors further show that the common central bank prefers national fiscal authorities’ cooperation in minimizing a union-wide welfare function that also includes price stability. Cavallari and Di Gioacchino (2005) show that fiscal authorities’ cooperation leads to favorable outcomes for output under demand/supply shocks and for inflation under demand shocks, while overall policy coordination improves macroeconomic stabilization only under demand shocks. The authors further show that monetary-fiscal symbiosis vanishes when there are other policy goals than cyclical stabilization, in particular costly policy instruments, as there must also be agreement on preferences’ weights. Della Posta and De Bonis (2009) reject the symbiosis result in the presence of asymmetric shocks, showing that policy coordination can be welfare improving even if the authorities have equal targets. Di Bartolomeo and Giuli (2011) also reject the symbiosis result in a closed-economy setting, when there is uncertainty about the effectiveness of the policy instruments.
Oros and Zimmer (2015) consider the impact of political (central bank) transparency on the policy mix in a monetary union under monetary policy transmission heterogeneity. The authors conclude that when the monetary transmission mechanism is relatively weak, higher monetary uncertainty may contribute to reduce inflation expectations, improving macroeconomic performance. Von Hagen and Mundschenk (2003) show that under strict inflation targeting the central bank controls union-wide output gap, while the national fiscal authorities determine the distribution of aggregate demand between them, engaging in a purely distributional game with inefficient outcomes unless policies are coordinated. Uhlig (2003) shows that in the absence of fiscal shocks and symmetrical countries size-wise, all fiscal authorities would be better off under a cooperative equilibrium characterized by a common fiscal policy of zero deficits. Ferre (2005) finds that in expansive phases of the economy, fiscal authorities’ cooperation leads to a higher deficit, while in Ferre (2008) the author shows that the non-cooperative case leads to a more volatile union-wide fiscal stance. Beetsma and Bovenberg (2005) consider the impact of fiscal authorities’ cooperation for the accumulation of debt, along with the interaction with structural distortions in labor markets. The authors find that ex-ante policy coordination among all the authorities can be beneficial. Furthermore, Acocella et. al. (2007b) show that fiscal authorities’ cooperation is beneficial when the labor market distortion is endogenously determined by trade union’s strategy, while in Acocella et. al. (2007a) fiscal leadership is desirable under a conservative central banker, rendering fiscal policies’ coordination preferable. Andersen (2005, 2008) shows that in the face of aggregate shocks, the fiscal authorities underestimate the monetary reaction, resulting in a more countercyclical fiscal policy, whereas in the case of idiosyncratic shocks, the monetary response is overestimated, and fiscal policy is insufficiently countercyclical. Gatti and Wijnbergen (2002) show that, in the event of adverse symmetric output (demand) shocks, the common central bank can impose fiscal authorities’ cooperation under the form of fiscal restraint by attaching a reward to the fiscal authorities in the form of a discretionary, nonstrategic level of the interest rate.

There is also a parallel literature that uses micro-founded dynamic stochastic general equilibrium (DSGE) models to examine optimal fiscal/monetary policies in a monetary union, where the focus is on transitional dynamics (see, e.g., Beetsma and Jensen, 2005; Gali and Monacelli, 2008; Ferrero, 2009). In a recent paper, Palek and Schwanebeck (2017) derive the welfare-maximizing (fiscal-monetary) policy response to demand and supply shocks in a two-country micro-founded DSGE monetary-union model with financial frictions, where the common nominal interest rate can directly affect inflation. The authors also allow for inflation to be directly affected by fiscal policy. Their analysis corresponds to that of full coordination of monetary and fiscal policies. Literally, the authors state: ‘…since we are interested in the output and inflation dynamics as well as the welfare losses arising from the cost channel, we do not take into account any strategic interaction between both policymakers’ (Palek and Schwanebeck, 2017, pp. 465-
6). On the contrary, we explicitly consider the strategic policy interactions in a monetary union when the two policy instruments can directly affect inflation. Their results are comparable to our regime of fiscal-monetary (overall) policy coordination. Naturally, we compare our results with the standard case in the literature that the two policy instruments cannot directly affect inflation, which makes them perfect substitutes in the stabilization process.

We can summarize our main results here: (i) the leader authority reacts to the follower authority’s reaction parameter, hence to the follower’s preference parameter, depending on the sign of a specific combination of structural parameters, (ii) the leader authority might choose not to trade-off its objectives (i.e., acting pro-cyclically), (iii) the symbiosis result collapses at the union level, too, as both the strategic and the fiscal regimes matter, (iv) the monetary authority chooses its preferable fiscal regime under simultaneous move according to the same before-mentioned combination of structural parameters, (v) the national fiscal authorities prefer to coordinate their policies under idiosyncratic shocks for all strategic regimes, (v) fiscal authorities’ cooperation can become welfare-improving, (vi) the simultaneous-move strategic regime may even emerge as superior to the leadership ones, and (vii) fiscal leadership with centralized fiscal policies can become a superior institutional arrangement even to overall policy coordination.

The next section presents the baseline model, while Section 3 describes the general solution at both the union and national levels for the alternative strategic regimes. In Section 4, we analyze the policy mix for all the alternative strategic and fiscal regimes. Section 5 proceeds to a welfare analysis for the two fiscal regimes, while Section 6 presents some results on the comparison among the alternative strategic regimes regarding union-wide pure cyclical macroeconomic stabilization. Finally, Section 7 concludes the paper.

2. The Model

We consider a monetary union consisted of two identical countries interconnected via traditional trade links and monetary policy. We model the monetary union as a closed area, assuming that both countries have no interconnections with countries outside the union. The model is a static representation of a reduced-form New Keynesian model based on an Aggregate Demand (AD) and a Phillips Curve (PC) equation, which constitutes a first-order approximation to a DSGE model with monopolistic competition and nominal rigidities (see, e.g., Gali, 2008). In particular, both equations can emerge from a micro-founded model that captures monopolistic competition in product and labor markets, along with sticky wages (see,

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7 This assumption is common in this literature. Moreover, the model includes various exogenous shocks that can be thought of as trade channels with countries outside the union.
e.g., Beetsma and Jensen, 2005; Gali and Monacelli, 2008). The static representation provides analytical results, which make the policy transmission mechanisms tractable and the study of the corresponding interactions manageable. This proves particularly useful in policy games, where a relatively simple analytical framework is required to allow comparisons of different solution concepts without resorting to numerical simulations. The model is mainly based on Andersen (2005, 2008) extended to include a cost channel of monetary policy, while it follows the same notation, too.

For each country $j$, the non-policy block of equations is given by:

$$
y_j = -\delta_r (i - \pi_j^e - \bar{r}_j) - \delta_r (\pi_j - \pi_k) + \delta_y y_k + \delta_g g_j + u_j
$$

$$
\pi_j = \omega_y y_j + \omega_g g_j + \omega_i i - \epsilon_j,
$$

where the index $k$ represents the other country. All variables represent log-deviations from long-run equilibrium values, apart from the decimal nominal interest rate, $i$. Thus, $\pi$ represents inflation, $y$ represents the output gap, while the variable $g$ represents fiscal policy, captured by the overall fiscal stance. We assume that before the shocks both economies have balanced budgets. The structural parameter $\bar{r}_j$ represents the long-run equilibrium real interest rate, which for simplicity we set equal to zero for both countries. The variables $u_j$ and $\epsilon_j$ are independently and identically distributed (i.i.d.-random) demand and supply shocks, respectively, with zero means and constant variances. We assume that they both are pure and uncorrelated. The inflation differential $\pi_j - \pi_k$ represents the real exchange rate and captures intra-union competitiveness (the terms-of-trade effect); in particular, higher prices for domestic products shift domestic demand to foreign. Finally, $\pi_j^e$ denotes the private sector’s (rational) expectation on country $j$’s future inflation.

Starting with the AD equation (1), all the parameters are positive. In particular, the parameter $\delta_r$ captures the real interest rate elasticity of aggregate demand, while $\delta_g$ captures the effectiveness of fiscal policy. The parameters $\delta_r$ and $\delta_y$ capture the interconnections between the two countries; in particular, the effect of competitiveness on domestic output and the relative openness of the economy, respectively (Ferre, 2008). The former corresponds to a cost spill-over effect, since higher domestic activity leads to higher prices and thus makes it possible for foreign partners to increase their market share, while the latter

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8 As quoted in Hughes Hallett et. al. (2011), Blanchard (2009, p. 27) calls for the “re-legalization of short-cuts and of simple models”, in order to improve intuition and communication.

9 This is a trivial assumption, as our model departs from debt considerations and focuses on stabilization policies. Thus, the model does not include an explicit government budget constraint. For a model with such a constraint, see, e.g., Beetsma and Bovenberg (1998).

10 As the two countries form a monetary union, the nominal exchange rate is fixed to unity, which means that the real exchange rate is equal to their price ratio.
corresponds to a demand spill-over effect, since a domestic fiscal expansion benefits trading partners by an increase in demand for foreign products. Both parameters are called trade externalities and may lead to insufficient stabilization (Andersen, 2005). The inflation differential works as a stabilization mechanism that compensates for the lack of an independent monetary policy. For example, if a country experiences a negative supply shock that increases inflation, a real appreciation would decrease exports to the other country (or to the rest of the union, in a multi-country setting), reducing output demand. This means that the country loses in competitiveness vis-a-vis the other country. However, the resulting reduction in demand eventually decreases inflation, thus acting as a stabilization mechanism (see, e.g., Landmann, 2012). Thus, the competitiveness channel works as a force of automatic stabilizers.11 The analysis does not change allowing for different consumption bundles across member-states (see, e.g., Andersen, 2005).

The existence of country-specific Phillips curves means that the ‘law of one price’ does not hold (see, e.g., Bofinger and Mayer, 2007). The PC equation (2) represents the short-run Lucas aggregate supply equation (see, e.g., Clarida et. al., 1999), which links country-specific inflation with the output gap.12 The former can be also directly affected by the two policy instruments, namely the domestic fiscal stance and the common nominal interest rate. Starting with the output gap, the parameter \( \omega_y \) is assumed to be positive, capturing nominal (price/wage) rigidities in the economy (see, e.g., Clarida et. al., 1999; Gali, 2008; Walsh, 2010). The existence of nominal rigidities provides a rationale for the monetary authority to influence output, as this passes through to inflation. The sign of the parameter for fiscal policy, namely \( \omega_g \), can be either positive or negative, capturing the direct effect that the plethora of the available fiscal instruments may have upon inflation (see, also, Chortareas and Mavrodimitrakis, 2016). Fiscal policy has a positive effect on output and through this a positive effect on inflation. However, following Andersen (2005, p. 5-6), it may also have (temporarily) separate effects on wage (price) inflation depending on the particular instrument used. For example, public expansions financed by value-added and excise taxes add (temporarily) to the inflationary pressure in the economy. However, it is also possible that tax increases may lead to wage moderation; in particular, high income taxes may increase labor supply causing a downward pressure on the wage rate (see, e.g., Baxter and King, 1993). According to Dixit and Lambertini (2003a), it can also arise via public investment or a production subsidy that raises private productivity, increasing the supply of goods.13 In order to capture standard reasoning on fiscal policy, we follow Andersen (2005, 2008) by assuming that \( \frac{\partial y_j}{\partial g_j} = \delta_g - \delta_r \omega_g > 0 \) and \( \frac{\partial \pi_j}{\partial g_j} = \omega_g + \omega_y \delta_g > 0 \).

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11 This procedure represents the adjustment of the real exchange rate through inflation differentials.

12 Following equation (2), country-specific inflation does not depend on expected inflation. We further examine this element on footnote 20.

13 Micro-foundations are provided by Gali and Monacelli (2008) for \( \omega_g < 0 \), and more recently by Palek and Scwanebeck (2017).
In our model, we also allow the common nominal interest rate to directly affect inflation, following the cost channel of monetary policy effect of Ravenna and Walsh (2006), where $\omega_t > 0$ (see, also, Walsh, 2010). The cost channel of monetary policy creates a meaningful policy trade-off for the central bank without the need for an exogenous cost-push shock. In particular, the authors assume a financial intermediary that monopolistically competitive firms must borrow from in order to pay for wages in advance. Thus, prices set by firms directly depend on the cost of borrowing, i.e. the loan rate; e.g., under a high (low) loan rate, prices will be also high (low). Financial intermediaries operate under perfect competition and the loan rate coincides with the basic interest rate set by the central bank. Moreover, as firms’ marginal cost is also a function of the loan rate, the PC equation depends also directly on the basic interest rate set by the central bank. This cost channel generates a meaningful trade-off between the output gap and inflation, as they will both fluctuate in response to supply and demand disturbances under the optimal policy. Thus, the authors provide theoretical justification for the fact that monetary policy directly affects the inflation adjustment equation if nominal interest rate movements directly affect real marginal costs, which has been empirically confirmed (see, e.g., Chowdhury et. al., 2006; Henzel et. al., 2009).

Similar to the assumption about fiscal policy, we assume that $\frac{\partial \pi_f}{\partial t} = \omega_t - \delta_r \omega_y < 0 \Rightarrow \delta_r \omega_y - \omega_t > 0$ in order to guarantee that monetary policy has the usual (expected) overall effect upon inflation, maintaining the nominal interest rate as a demand-side policy instrument.

Alternative explanations for the (positive) direct effect of the nominal interest rate on inflation are provided by Ismihan and Ozkan (2012) and De Grauwe (2012). The former authors assume that the level of total bank credits affects output supply in a Barro-Gordon (1983) framework, where total bank credits are negatively affected by the loan rate set by a monopolistically-competitive commercial bank. Thus, the loan rate directly affects inflation in a positive manner. De Grauwe (2012) let asset (stock) prices to affect both the aggregate demand and supply of a behavioral macroeconomic model. Regarding aggregate supply, an increase in stock prices makes external risk premia to decrease, reducing firms’ credit costs. As the nominal interest rate affects negatively the stock prices, the former would directly affect inflation in a positive manner.

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14 For the importance of the exogenous cost-push shock in creating a meaningful policy trade-off in the standard New Keynesian model, see Gali (2008, Chapter 5).
15 This means that there is complete interest rate pass-through. Kobayashi (2008) examines the case of incomplete interest rate pass-through resulting from real and nominal frictions in financial markets.
16 De Grauwe (2012) proposes a behavioral macroeconomic model that modifies the standard New Keynesian model mainly in two aspects: (i) it encompasses inertia on the output gap and the inflation rate by incorporating backward-looking elements, too, and (ii) it departs from rational expectations for forward-looking variables, introducing heuristics, instead.
17 Areosa and Areosa (2016) introduce an inequality channel through which the real interest rate can affect the PC equation in an otherwise standard New Keynesian DSGE model, by incorporating unskilled agents with no access to financial markets. The authors show that if there is an excess of unskilled agents, inequality rises with the interest rate and increases inflation. This means that there is a direct positive link between the nominal interest rate and inflation through the inequality channel.
In a recent paper, Palek and Schwanebeck (2017) examine optimal fiscal-monetary policy in a monetary union under financial frictions in a two-country DSGE model, assuming a cost channel of monetary policy, following Ravenna and Walsh (2006). The authors provide micro-foundations for the country-specific Phillips curve (eq. 2), where both policy instruments can directly affect inflation. They are interested in the output and inflation dynamics, as well as in the welfare losses arising from the cost channel of monetary policy. However, the authors do not impose our assumption $\frac{\partial \pi_f}{\partial i} < 0$, allowing the common nominal interest rate to become a supply-side policy instrument. Moreover, they also consider heterogeneous financial frictions between the two member-states. In our model, this can be captured by asymmetric supply shocks between the two member-states, as we only deal with discretionary policies.

We can find the descriptive non-policy block of equations at the union level by averaging the country-specific equations (1)-(2). We get:

$$ y = \frac{1}{1-\delta_y} (-\delta_r i + \delta_g g + u) $$

$$ \pi = \omega_y y + \omega_g g + \omega_i i - \varepsilon, $$

where for every variable $x$, it follows that $x = \frac{1}{2} (x_j + x_k)$. The policy instruments for the national fiscal authorities and the monetary authority are $g_j, g_k$ and $i$, respectively.

Following Kydland and Prescott (1977) and Barro and Gordon (1983), we assume that all the authorities have complete control over a policy instrument and preferences over some variables that can be approximated by a quadratic loss function. This methodology is standard in this literature and it is based on Theil’s (1956) flexible target approach, where a policymaker minimizes the inevitable deviations of some targets in the form of a quadratic objective (loss) function under the economy’s constraints. The quadratic loss function illustrates that objectives are symmetric; i.e., the authorities weight the same either a positive or a negative deviation of a concerned variable from a target value.\(^{18}\)

The authorities’ loss functions are given by:

$$ L_M = \frac{1}{2} (\pi^2 + a_M y^2) $$

$$ L_{F_j} = \frac{1}{2} (g_j^2 + a_{F_j} y_j^2). $$

\(^{18}\)Following Acocella et. al. (2013), quadratic functions are used both because they are mathematically tractable and because they encompass useful economic properties. As deviations from the target are associated with increasing costs, the marginal rate of substitution between any two target variables is never constant, depending on the values of the two variables at the specific point it is computed. Moreover, quadratic forms can be obtained as second-order approximations of more complex functions (see, e.g., Woodford, 2003).
where ‘M’ stands for the ‘Monetary’ authority and ‘F’ for the national ‘Fiscal’ authorities. They all target long-run equilibrium values of concerned variables, assumed to equal zero; i.e., they all seek to minimize deviations of their concerned variables from long-run equilibrium. This means that they agree on the steady state of overall optimal policy (Uhlig, 2003), hence they have non-conflicting objectives. In particular, we assume that the national fiscal authorities share identical preferences and that they are concerned with the output gap and the deviation from the balanced budget, whereas the common central bank is concerned with the average output gap and inflation in the union.

The parameters $a_M$ and $a_F$ are both positive and represent the weights that the authorities place on output-gap stabilization, meaning the monetary authority and the fiscal authorities, respectively. These weights are relative to the preference parameters for inflation and the fiscal stance, respectively, which for simplicity we have both set equal to unity. Regarding monetary authority, the larger $a_M$, the more flexible is the inflation targeting approach that the common central bank follows (Svensson, 1997). Naturally, $a_M < 1.0$. The case of $a_M = 0$ corresponds to strict inflation targeting, where the common central banker is only concerned with stabilizing union-wide inflation.

The specification of the monetary and the fiscal authorities’ loss functions, namely equations (5) and (6), follows Uhlig (2003) and Andersen (2008). This set of loss functions represents a realistic mapping of the actual policy-making concerns in the EMU (see, also, Chortareas and Mavrodimitrakis, 2017). Following the most recent literature, we include each country’s fiscal stance in the fiscal authorities’ loss functions, as countries in the EMU are constrained by both the SGP and the FC. Thus, the fiscal stance is simultaneously a target and an instrument for the national fiscal authorities. However, fiscal policymakers are not directly concerned about inflation, since the task of controlling inflation is delegated to the common central bank. However, the inclusion of a terms-of-trade effect in the aggregate demand equation creates an implicit preference for inflation stabilization for the national fiscal authorities (see, e.g., Andersen, 2005, 2008). Andersen and Spange (2006) show that equation (6) can be derived from a representative household’s utility function that depends positively on the private consumption bundle and on the provision of public goods and negatively on labor supply, where the private consumption bundle is defined over the consumption of the domestic and the foreign commodity.

The main objective of this paper is to investigate the macroeconomic policy mix in the monetary union we have just described that arises from the interaction between the common central bank and the two national fiscal authorities when the two policy instruments can also directly affect the PC equation and when the authorities have non-conflicting but different objectives under alternative assumptions about the ordering of moves. The ordering of moves represents the institutional setting in the monetary union where policies are being implemented. We analyze the standard one-shot policy games of simultaneous move,
fiscal and monetary leadership. In all scenarios, the time context begins with the private sector forming expectations about future inflation rationally and not strategically (Uhlig, 2003); then, demand and supply shocks are realized; finally, the authorities choose their control instrument in order to achieve their goals according to the particular institutional setting (strategic regime), hence considering discretionary policies. The strategic regime of simultaneous move demands all the authorities to act independently and simultaneously, where the equilibrium is described by a Cournot-Nash equilibrium. For the two leadership regimes we assume that the authority that has the lead makes its move before the follower authority, while it takes into account the way the latter will react to its choice of the policy instrument. These Stackelberg games are solved using backward induction and the equilibrium rests on sub-game perfection. The fiscal leadership regime requires the two fiscal authorities to lead the game with the common central bank, while in the monetary leadership regime the monetary authority has the lead and the national fiscal authorities follow. It follows that policies are time-consistent, hence $\pi_f^e = \pi^e = 0$ (see, e.g., Uhlig, 2003; Andersen, 2008; among others). In any case, we assume that the national fiscal authorities move simultaneously.

The model also assumes that there is no uncertainty about structural parameters between the two fiscal authorities and between them and the monetary authority.

In every strategic regime we also consider the case of fiscal authorities’ cooperation, where the national fiscal authorities minimize a joint loss function according to a straightforward utilitarian criterion that corresponds to simply averaging the two loss functions given by equation (6). This is common in all papers in the literature that consider an interconnection between the countries that form a monetary union (see, e.g., Debrun, 2000; Dixit and Lambertini, 2001, 2003a; Cavallari and Di Gioacchino, 2005; Ferre, 2008, 2012; Andersen, 2005, 2008; among others). The joint loss function is given by:

$$L_F = \frac{1}{2} \left( L_{F_j} + L_{F_k} \right) = \frac{1}{4} \left[ g_j^2 + g_k^2 + \alpha_F (y_j^2 + y_k^2) \right]$$

We also investigate the case of fiscal-monetary (overall) policy coordination, where the monetary authority and the two national fiscal authorities choose their policy instruments so as to achieve their joint objectives. In particular, we create a loss function that is the sum of each authority’s loss function, namely:

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19 For a novel framework that generalizes the time structure through players’ rational inattention that creates rigidities in the timing of moves and makes the game more dynamic and asynchronous, see Libich and Stehlik (2010).

20 We have already incorporated this result in equation (4). It needs rational expectations on the part of the private sector, the private sector to form its expectations prior to the shocks’ realization, and all the authorities to target long-run equilibrium values, in order for policies to be time-consistent. We follow Uhlig (2003) by using this result at the beginning, rather than deriving it as the last step of the calculation.

21 For models where the national fiscal authorities in a monetary union do not move simultaneously but sequentially, see Chortareas and Mavrodimitrakis (2016, 2017).

22 We can think of this game as a regime that is in place for long horizon; then, repeated play of this game would reveal the exact structural parameters (Lane, 2003).
\[ L_{OC} = L_M + \frac{1}{2} (L_{F_j} + L_{F_k}) = \frac{1}{2} \left[ \pi^2 + \frac{1}{2} (g_j^2 + g_k^2) + a_M y^2 + \frac{1}{2} a_F (y_j^2 + y_k^2) \right], \]

where ‘OC’ stands for ‘Overall Coordination’. We follow Flotho (2012) by adding the mean of the fiscal authorities’ loss functions. Naturally, this joint loss function includes both union-wide and country-specific variables, while all spillover effects are fully internalized.

We also define the social planner’s loss function by assuming that it encompasses the union-wide variables that the authorities are concerned with. For this specification, we follow Beetsma and Bovenberg (1998) and mainly Andersen (2008). We assume the following social loss function\(^{23}\) for the monetary union:

\[ L_S = \frac{1}{2} (\pi^2 + b_S g^2 + a_S y^2), \]

where ‘S’ stands for ‘Society’ (or the ‘Social planner’), and \(a_S\) and \(b_S\) are the weights that society places upon union-wide output gap and fiscal stance, respectively, relative to inflation. Thus, the social planner minimizes equation (9) subject to the non-policy block of equations for the monetary union, namely equations (3) and (4). Andersen (2008) uses the above loss function in order to examine fiscal-monetary (overall) policy coordination in a monetary union, as the two cases deliver the same equilibrium solutions for the union-wide macroeconomic variables. Equation (9) can be also used as a welfare criterion for the comparison of the alternative strategic regimes.

We conclude this section by computing the reduced form country-specific aggregate demand equations with respect to the policy instruments and shocks.\(^{24}\) We end up with:

\[ y_j = -Z_i l + Z_g g_j + Z_g^* g_k + Z_\varepsilon (\varepsilon_j - \varepsilon_k) + Z_u u_j + Z_u^* u_k, \]

\[ Z_i = \left[ \frac{\partial y_j}{\partial l} \right]_{1-\delta_y}, \quad Z_g = \left[ \frac{\partial y_j}{\partial g_j} \right]_{1-\delta_y}, \quad Z_g^* = \left[ \frac{\partial y_j}{\partial g_k} \right]_{1-\delta_y}, \quad Z_\varepsilon = \left[ \frac{\partial y_j}{\partial \varepsilon_{k-j}} \right]_{1-\delta_y}, \quad Z_u = \left[ \frac{\partial y_j}{\partial u_j} \right]_{1+\delta_y}, \quad Z_u^* = \left[ \frac{\partial y_j}{\partial u_k} \right]_{1+\delta_y}. \]

Equation (10) is a reduced form equation that defines a target variable, namely country-specific output demand, with respect to the policy instruments and exogenous shocks. All the \(Z\) parameters are country-specific output demand elasticities relative to the three policy instruments \((Z_i, Z_g, Z_g^*)\), to domestic and foreign demand shocks \((Z_u, Z_u^*)\) and to supply shocks’ asymmetries \((Z_\varepsilon)\), where the latter is presented in absolute terms and

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\(^{23}\) Similar loss functions can be also found in DSGE models that examine optimal fiscal-monetary policies in a monetary union, such as Gali and Monacelli (2008), Ferrero (2009), and more recently Palek and Schwanebeck (2017). In all models, society’s loss function is derived from the representative household’s utility function following the methodology of Woodford (2003). In the EMU context, we can think of the social planner as the European Commission.

\(^{24}\) We solve together the two aggregate demand equations (eq. 1) for both countries. We then subtract the two PC equations (eq. 2) to create \(\pi_j - \pi_k\), and we incorporate the latter to both the aggregate demand equations, which we solve together.
defined as $\varepsilon_j - \varepsilon_k$. It is straightforward that under $1 - \delta_y > 0$, all the $Z$ parameters are positive. The corresponding ones that refer to the policy instruments define policy effectiveness, as long as they are different to zero. The importance of the interconnections for those elasticities is profound. First, domestic output demand is directly affected by foreign demand shocks, by supply shocks’ asymmetries and by foreign fiscal policy only through the interconnections; in the opposite case of $\delta_y = \delta_e = 0$, domestic aggregate demand is only affected by domestic demand shocks, while supply shocks’ asymmetries are present only because of the terms-of-trade effect. Second, both domestic and foreign fiscal policy affect domestic aggregate demand. The former’s direct effect is positive, while it affects it negatively through the terms-of-trade effect. The foreign fiscal policy affects positively domestic output through both the trade and the terms-of-trade effects. Third, all the above elasticities are independent of the cost channel of monetary policy, as it is assumed to be the same for the two countries. This means that it does not affect the terms-of-trade effect and thus it cannot affect aggregate demand.25

3. The General Solution at the Union Level

The monetary authority and the national fiscal authorities have two targets but only one instrument: (i) the monetary authority controls the common nominal interest rate, $i$, to minimize its loss function (eq. 5), and (ii) each national fiscal authority controls its fiscal stance, $g_j$, in order to minimize its loss function (eq. 6). Both problems follow Theil’s (1956) flexible target approach. Each authority chooses its instrument of control by equating the marginal rate of transformation with the marginal rate of substitution between the two target variables, where the latter is also based on the authorities’ preference parameters, $a_M$ and $a_F$. Each authority’s problem ends up with a corresponding policy rule that combines the concerned macroeconomic variables.

The country-specific fiscal rule for the national fiscal authorities under decentralization is given by:

$$g_j = -\phi_{g_j}y_j,$$

(11)

where $\phi_{g_j} = a_F \frac{dy_j}{dg_j}$ is the country-specific fiscal reaction parameter. The symmetry assumption for the two member-states ensures that $\frac{dy_j}{dg_j} = \frac{dy_k}{dg_k}$, which leads to the two fiscal rules being symmetric, too, meaning

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25 Supply shocks’ asymmetries could also represent a cost channel heterogeneity between the two member-states, reflecting possible differences in the degree of competition in financial markets. In this sense, the PC would depend on the loan rate, $i_l$, as $i_l = i + v_j$, where $v_j$ is an i.i.d. cost-push shock with zero mean and constant variance. Then, it follows that $\varepsilon_j = -\omega_l v_j$ and $\varepsilon_j - \varepsilon_k = -\omega_l (v_j - v_k)$. 

\( \phi_{gj} = \phi_{gk} \). For the centralized case, where both fiscal authorities minimize equation (7), the first order condition becomes:

\[
g_j + a_F \left( \frac{dy_j}{dg_j} y_j + \frac{dy_k}{dg_j} y_k \right) = 0
\]  

(12)

At the union level, the two rules can be found to be:

\[
\text{MR}: y = -\phi_\pi \pi
\]  

(13)

\[
\text{FR}: g = -\phi_g y,
\]  

(14)

where ‘MR’ stands for ‘Monetary Rule’ and ‘FR’ for ‘Fiscal Rule’, and \( \phi_\pi = \frac{1}{a_M} \frac{d\pi}{d\epsilon} \). The parameters \( \phi_\pi \) and \( \phi_g \) correspond to the monetary and the (union-wide) fiscal reaction parameters, respectively. For the decentralized fiscal regime, it is straightforward that \( \phi_{gj} = \phi_g \), while for the centralized case, we get \( \phi_g = a_F \left( \frac{dy_j}{dg_j} + \frac{dy_k}{dg_k} \right) \). Both rules represent closed-form equilibrium solutions that show how both monetary and union-wide fiscal policy react to a change in the authorities’ concerned macroeconomic variables. Both reaction parameters are functions of the model’s structural \((\delta_r, \delta_t, \delta_y, \delta_g, \omega_y, \omega_i, \omega_g)\) and preference \((a_M, a_F)\) parameters, while they can be of either sign; in particular, a possible positive sign defines a trade-off between the authorities’ target variables. They depend upon the ordering of moves, namely the three strategic regimes of fiscal/monetary leadership and simultaneous move, and also on whether fiscal policies are coordinated or not.

At the union level, the two descriptive equations, namely the AD equation (3) and the PC equation (4), along with the monetary rule (eq. 13) and the fiscal rule (eq. 14) create a 4 * 4 system of (log)-linear equations, with unknowns the inflation rate, the output gap, the fiscal stance and the common nominal interest rate. The two former variables represent the target variables, while the two latter the policy instruments, although country-specific fiscal stances are both targets and instruments, following the fiscal authorities’ loss functions (eq. 6). This assumption is responsible for the policy conflict (see, e.g., Dixit and Lambertini, 2003a; Kempf and von Thadden, 2013). Solving all four equations simultaneously, we end up with the following equilibrium solutions:

\[
\pi = \frac{1}{\Omega} (\omega_i u - \delta_r \epsilon)
\]  

(15)

\[
y = -\frac{\phi_\pi}{\Omega} (\omega_i u - \delta_r \epsilon)
\]  

(16)
\[ g = \frac{\phi_g \phi}{\Omega} (\omega_i u - \delta_r \varepsilon) \]  
(17)

\[ i = \frac{[1 + (\omega_y - \omega_g \phi_g) \phi] u - (1 - \delta_y + \delta_g \phi_g) \phi \pi \varepsilon}{\Omega}, \]  
(18)

where:

\[ \Omega = \delta_r [1 + (\omega_y - \omega_g \phi_g) \phi] - (1 - \delta_y + \delta_g \phi_g) \omega_i \phi = \delta_r + [\omega_y \delta_r - (1 - \delta_y) \omega_i] \phi \pi - (\delta_g \omega_i + \delta_r \omega_g) \phi \pi \]  
(19)

We call \( \Omega \) (eq. 19) the ‘reference parameter’, as it ‘refers’ to a particular institutional arrangement; thus, it captures differences on equilibrium solutions of the union-wide macroeconomic variables across strategic and fiscal regimes. Following the union-wide equilibrium solutions, namely equations (15)-(18), we can extract some important remarks.

**Remark 1:** The cost channel of monetary policy makes union-wide (pure) demand shocks not to be fully stabilized at the union level.

The vast literature does not take into consideration the cost channel of monetary policy. In this special case, union-wide pure demand shocks are fully stabilized at the union level. Countering pure demand shocks pushes both the output gap and inflation in the same direction, as there is no trade-off between those two. Thus, the monetary authority succeeds in fully-stabilizing pure demand shocks (\( i = \frac{1}{\delta_r} u \)) and the union-wide fiscal stance is passive.\(^\text{26}\) This is the ‘divine coincidence’ property of the standard closed-economy New Keynesian model (Blanchard and Gali, 2007), which illustrates the optimality of the strict inflation targeting monetary policy framework (see, also, Clarida et. al, 1999). The irrelevance of demand shocks at the union level in a micro-founded monetary union model is also demonstrated by Beetsma and Jensen (2005). The existence of the cost channel of monetary policy, which mainly intends to capture the existence of a financial sector in the economy in the most simple way, makes demand shocks not to be fully stabilized at the union level by creating a trade-off between inflation and output gap, even in the absence of supply shocks, similar to Ravenna and Walsh (2006) (see, also, Palek and Schwanebeck, 2017). In this case, the

\(^\text{26}\) However, this does not mean that the country-specific fiscal policies are passive. It only means that their reactions either cancel out at the union level or are countered by the monetary authority.
union-wide fiscal stance is not passive, which means that the national fiscal authorities supplement the monetary authority in the stabilization process.

Andersen (2008) considers shocks that are not pure, in the sense that they can simultaneously affect demand and supply in various ways. However, pure demand shocks emerge as a special case, where his result does not differ from the literature. Cavallari and Di Gioacchino (2005), Lambertini and Rovelli (2004) and Oros and Zimmer (2015) contrast from the literature in this aspect, as they all assume interest-rate smoothing on the part of the monetary authority, which corresponds to the inclusion of the square of the common nominal interest rate in the monetary authority’s loss function. Because of interest-rate smoothing, the monetary reaction to shocks is milder, leaving demand shocks partially stabilized.

**Remark 2:** At the union level, all macroeconomic variables are affected by union-wide demand and supply shocks and not by shocks’ asymmetries. Thus, idiosyncratic shocks are fully stabilized at the union level.

Following equations (17) and (18), the two policy instruments at the union level do not react to shocks’ asymmetries. In spite of the ordering of moves, we will see that the two national fiscal authorities respond to this shocks in exactly the opposite way, as the two countries are identical; hence, their responses cancel out at the union level. The common central bank reacts neither in the simultaneous move nor in the fiscal leadership regime, as there is no average shock to the monetary union, whereas in the case of monetary leadership it does not react because it anticipates that the reactions of the fiscal authorities will be offset. In a previous paper, we have shown that asymmetric demand shocks pass through to the union-wide macroeconomic variables when the two national fiscal authorities follow a sequential game, hence they do not move simultaneously, and fiscal policy can directly affect inflation (Chortareas and Mavrodimitrakis, 2016).

**Remark 3:** If the two policy instruments are not perfect substitutes in the stabilization process, then the deficit bias result does not hold and fiscal policy becomes non-neutral at the union level.

The standard case in the vast literature that corresponds to a special case in our model is when the two policy instruments do not directly affect inflation, which means that they are perfect substitutes in the

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27 For an interest-rate smoothing central bank in a closed-economy setting, see, e.g., Buti et. al. (2001).
stabilization process. The fiscal reaction parameter does not affect the reference parameter (eq. 19) and thus neither equilibrium inflation nor the output gap, given by equations (15) and (16), respectively. However, it affects both the union-wide fiscal stance and the common nominal interest rate for supply shocks, following equations (17) and (18). This result holds for all strategic regimes, and it is known in the literature as the deficit-bias result (see, e.g., Beetsma and Bovenberg, 1998; Buti et. al., 2001; Uhlig, 2003; among others). Naturally, the fiscal reaction parameter is a function of the fiscal authorities’ preference parameter, \(a_F\). Thus, the fact that the fiscal authorities’ preference parameter cannot affect either the equilibrium union-wide output gap or inflation corresponds to (endogenous) policy neutrality on the part of the (union-wide) fiscal policy. In general, the fiscal reaction parameter needs at least one policy instrument to directly affect the PC equation in order to affect the reference parameter, \(\Omega\), and thus affecting equilibrium inflation and the output gap (see, e.g., Debrun, 2000; Andersen, 2005, 2008).

Following equation (19), the deficit bias result can also be obtained under \(\delta_g \omega_i + \delta_r \omega_g = \delta_g \omega_i + (-\delta_r) \omega_g = 0 \Rightarrow \frac{\delta_g}{(1-\delta_y)\omega_g} = \frac{-\delta_r}{(1-\delta_y)\omega_i}\) for \(\omega_g, \omega_i, 1-\delta_y \neq 0\), where each ratio represents the analogy of the (direct) impact that each policy instrument has upon the two target variables, following equations (3) and (4). Each ratio gives the gradient (i.e., the marginal rate of transformation) of the output gap and inflation as a result of changes in each policy instrument. If the two ratios are equal, which only holds under \(\omega_g < 0\), the two policy instruments are again perfect substitutes in the stabilization process, along with the previous case of \(\omega_g = \omega_i = 0\). The comparison of the two ratios \(\frac{\delta_g}{\omega_g}\) and \(\frac{-\delta_r}{\omega_i}\) reveals the more efficient policy instrument according to output gap (versus inflation) stabilization. We thus proceed with the following definition.

**Definition:** We define fiscal policy’s relative efficiency in stabilizing aggregate demand (relative to inflation) following the sign of \(\delta_g \omega_i + \delta_r \omega_g\), when \(\omega_g < 0\). In particular, (i) if \(\delta_g \omega_i + \delta_r \omega_g > 0\), then fiscal policy is more (less) efficient in stabilizing aggregate demand (relative to inflation) than monetary

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28 It is straightforward, combining the union-wide PC equation (4) with the monetary rule (eq. 13), after setting \(\omega_g = \omega_i = 0\).
29 Agell et. al. (1996) establish the deficit (spending) bias result in a small open economy under an activist fiscal policy and a monetary policy exclusively committed to price stability.
30 Endogenous policy neutrality with respect to a target variable is present if the optimal value of such a variable is not affected by any change in the policymaker’s preferences. Endogeneity refers to the flexible target approach (see Acocella et. al., 2013, p. 23).
31 A similar remark is made by Onorante (2004), where monetary policy is assumed to be relatively more efficient on prices than fiscal policies, which means that monetary policy has a comparative advantage in controlling prices. In her model, the money supply is assumed to be the monetary instrument, where it affects positively the price level and negatively the unemployment rate, similar to fiscal policy.
policy,\textsuperscript{32} and (ii) if $\delta_g \omega_i + \delta_r \omega_g = 0$, then fiscal policy is equally efficient in stabilizing aggregate demand (relative to inflation) with monetary policy. In the latter case, the two policy instruments are perfect substitutes in the stabilization process, along with the case of $\omega_g = \omega_i = 0$.

4. The Policy Mix under Alternative Institutional (Strategic) Regimes

In this section we present and analyze the solutions for the fiscal and monetary reaction parameters for all strategic and fiscal regimes, along with the reference parameters, in order to understand the policy mix in the monetary union. The results are shown in Table 1.\textsuperscript{33}

[Insert Table 1]

4.1 The Authorities’ Reaction Parameters

The two strategic regimes of simultaneous move and fiscal leadership deliver the same monetary reaction parameter for both fiscal regimes, which is unambiguously positive. The common central bank faces a trade-off between inflation and output gap at the union level, hence pursuing a ‘lean against the wind’ monetary policy (see, e.g., Clarida et. al., 1999). In particular, the monetary authority reacts to a possible rise (fall) in the union-wide inflation rate caused by an average negative (positive) supply shock by reducing (increasing) the union-wide output gap. In order to do that, it raises (decreases) the common nominal interest rate. Its reaction is stronger the larger the slope of the PC equation ($\omega_y$), the lower the weight that it assigns to output-gap stabilization ($a_M$) and the lower the cost channel of monetary policy ($\omega_i$), where the latter makes the monetary authority less reactionary (Palek and Schwanebeck, 2017).

However, both the trade effect, $\delta_y$, and the semi-elasticity of the interest rate, $\delta_r$, now affect the monetary reaction parameter positively, reducing the cost channel effect of monetary policy. In the standard case in the literature where there is no cost channel of monetary policy, the monetary reaction parameter becomes $\phi_{\pi M} = \frac{\omega_y}{a_M}$ (see, e.g., Dixit and Lambertini, 2003a; Uhlig, 2003; Ferre, 2005; Andersen, 2008; Flotho, 2012;\textsuperscript{32} The second case of $\delta_g \omega_i + \delta_r \omega_g < 0$ holds under $\omega_g < 0$ and $|\omega_g| > \frac{\delta_g \omega_i}{\delta_r}$. However, we have already impose two restrictions for the values of $\omega_g$ and $\omega_i$ in our model, namely $\omega_g + \omega_r \delta_g > 0$ and $\omega_i - \delta_r \omega_y < 0$, respectively. This means that in the case of $\omega_g < 0$, its absolute value must not exceed $\omega_y \delta_g$, where $\omega_y > \frac{\omega_i}{\delta_r}$; hence, $\frac{\delta_g \omega_i}{\delta_r} < \omega_y \delta_g$. Thus, the special case of $\delta_g \omega_i + \delta_r \omega_g < 0$ can be satisfied under our parameter values' restrictions for $\omega_g < 0$ and $\frac{\delta_g \omega_i}{\delta_r} < |\omega_g| < \omega_y \delta_g$, where $\omega_y > \frac{\omega_i}{\delta_r}$ holds by assumption.\textsuperscript{33} See Appendix A for details on the construction of Table 1.

~ 20 ~
Chortareas and Mavrodimitrakis, 2016, 2017; among others). In this case, there is no trade-off between union-wide inflation and the output gap without supply shocks. The existence of the cost channel of monetary policy creates a trade-off to the monetary authority even when there are no supply shocks and fiscal policy does not directly affect inflation (Ravenna and Walsh, 2006). Moreover, the monetary reaction parameter differs from the one for the overall policy coordination regime only in the weight that the corresponding authorities who set policy place on output-gap stabilization relative to inflation. Naturally, for this strategic regime, monetary policy would be less reactionary.

The determination of the fiscal reaction parameter’s sign for the overall policy coordination regime depends on the sign of $\delta_g \omega_i + \delta_r \omega_g$. If it is positive (negative), the fiscal reaction parameter is unambiguously negative (positive), which means that union-wide fiscal policy reacts pro-(counter)-cyclically. In this case, $\frac{\partial \phi_g}{\partial \phi} > 0$ ($\frac{\partial \phi_g}{\partial \phi} < 0$), which means that union-wide fiscal policy supplements (substitutes for) monetary policy in the stabilization process. Moreover, the higher the authorities’ joint weight on output-gap stabilization and/or the cost channel of monetary policy, the stronger is the pro-(counter)-cyclicality of fiscal policy. If $\delta_g \omega_i + \delta_r \omega_g = 0$, meaning that the two policy instruments are perfect substitutes in the stabilization process, the union-wide fiscal policy is passive and monetary policy takes all the burden of stabilizing the cycle. Both the reaction parameters for the overall policy coordination regime do not depend on the terms-of-trade effect, $\delta_r$, as the latter is not being exploited by the fiscal authorities when they cooperate.

For the simultaneous-move strategic regime, the union-wide fiscal policy reacts unambiguously counter-cyclically for both fiscal regimes, contradicting with the previous case of overall policy coordination. In the decentralized case, the fiscal reaction parameter does not depend on the monetary reaction parameter, hence neither on the cost channel of monetary policy, $\omega_i$, nor on the real interest rate semi-elasticity of aggregate demand. However, it depends positively on the (actual) fiscal multiplier, $Z_g$, which, following the country-specific aggregate demand equation (10), is substantially affected by both interconnections. In the case of overall policy coordination, all the authorities cooperate with each other so as not to exploit the terms of trade. However, under decentralized fiscal policies, each fiscal authority tries to exploit the terms-of-trade effect to gain in competitiveness vis-a-vis the other country. This channel works through fiscal policy and also through its direct impact upon inflation. In the special case where the latter channel does not exist, i.e. under $\omega_g = 0$, the terms-of-trade effect affects the fiscal reaction parameter

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34 It is also exactly the same with the classical reference of Clarida et. al. (1999) for monetary policy analysis. Andersen (2005) and Ferre (2008, 2012) consider strict inflation targeting, instead.

35 Our results are equivalent to Flotho (2012) for $\omega_i = 0$ and $\omega_g < 0$: the union-wide fiscal policy reacts unambiguously counter-cyclically, whereas under $\omega_g = 0$, fiscal policy is passive.
negatively, as it works as an automatic stabilizer (see, e.g., Landmann, 2012). If the two countries in the monetary union are not inter-connected, the fiscal reaction parameter equals the effectiveness of fiscal policy, $\delta_g$, multiplied by the weight, $a_F$ (see, e.g., Lambertini and Rovelli, 2004; Ferre, 2005; Cavallari and Di Gioacchino, 2005).\footnote{Cavallari and Di Gioacchino (2005) consider a fiscal spill-over effect between the two countries following Dixit and Lambertini (2003a), which enhances the horizontal coordination problem between the two fiscal authorities. However, it cannot affect the fiscal reaction parameter for the decentralized case, as each fiscal authority takes the other authority's fiscal stance as given. See, also, Oros and Zimmer (2015). This is what Kempf and von Thadden (2013) mean by (in-)significant direct (fiscal) spill-overs.}

In the decentralized case, the national fiscal authorities change their fiscal stances only in response to their own output gap, whereas in the centralized case, following equation (12), they also react to changes in the other country’s output gap, although in a countercyclical manner.\footnote{See equation (A.1) in the Appendix A for the country-specific fiscal rule.} In the latter case, the externalities created by the interconnections between the two countries are internalized (see, e.g., Uhlig, 2003; Andersen, 2005). At the union level, the fiscal reaction parameter equals the union-wide fiscal multiplier, $\frac{\delta_g}{1-\delta_y}$, multiplied by the authorities’ weight on output-gap stabilization, $a_F$. If the fiscal authorities cooperate, their responses to output changes cancel out, so their joint reaction has the same result with the effect of fiscal policy upon the union-wide output gap.

It is straightforward that the fiscal reaction parameter for the centralized case is higher than the one for the decentralized case, as $\phi^{SM}_{g} - \phi^{SM}_{gnc} = a_F Z_g^* > 0$, which means that (union-wide) fiscal policy is more counter-cyclical under fiscal cooperation. This further implies that the decentralized case leads to insufficient stabilization at the country level. However, the case of fiscal policies’ coordination departs further from the fiscal-monetary (overall) policy coordination regime. By cooperating with each other, the national fiscal authorities succeed in strengthening their strategic position relative to the common central bank (see, e.g., Beetsma and Bovenberg, 1998; Cavallari and Di Gioacchino, 2005). In complete contrast, under the decentralized case, their attempt to exploit one another is completely inefficient, weakening their strategic position, which diminishes under the case of fiscal-monetary (overall) policy coordination.

We proceed to the leadership strategic regimes. Under monetary leadership, the fiscal reaction parameter is exactly the same with the simultaneous-move strategic regime, for both fiscal regimes. On the contrary, the monetary reaction parameter differs in depending on the fiscal reaction parameter. However, this takes place only under $\delta_g \omega_i + \delta_r \omega_g \neq 0$; otherwise, the monetary reaction parameter is equal to the one from the simultaneous-move strategic regime, i.e. $\phi^{ML}_\pi = \phi^{SM}_\pi$.\footnote{See also Kirsanova et. al. (2005) and Flotho (2012) for both the simultaneous move and the monetary leadership regimes.}
For the fiscal leadership strategic regime,\(^{39}\) the national fiscal authorities take into account the monetary authority’s reaction function, where the parameter \(V_i\) defines the reaction of the common nominal interest rate to a possible change in the average fiscal stance, i.e. \(V_i = \frac{\partial i}{\partial g}\). Its sign is unambiguously positive under \(1 - \delta_y > 0\), which means that the monetary authority reacts counter-cyclically. Thus, the sign of the fiscal reaction parameter for the decentralized fiscal regime cannot be determined; in particular, fiscal policy can be either counter-cyclical or pro-cyclical, depending on the sign of \(Z_g - \frac{1}{2}Z_i V_i\). In general, it is straightforward that the fiscal reaction parameter for the fiscal leadership regime is lower than the one for the simultaneous-move regime, as \(\phi_{\delta nc}^{SM} - \phi_{\delta nc}^{FL} = \frac{1}{2}Z_i V_i > 0\). Under fiscal leadership, the national fiscal authorities anticipate the monetary reaction, hence becoming less countercyclical. If the fiscal authorities anticipate that the impact of fiscal policy on country-specific output gap will be larger than the one of the monetary policy response, then the fiscal reaction parameter will be positive and fiscal policy will be unambiguously counter-cyclical. In this case, the monetary policy’s counteraction cannot overturn the counter-cyclical nature of fiscal policy. In the opposite case, fiscal policy is pro-cyclical; i.e., the fiscal authorities anticipate that the monetary response will be too strong, so they move pro-cyclically to induce a counter-cyclical overall reaction. In the special case of \(\omega_t = \omega_g = 0\), then \(V_i = \frac{\delta_g}{\delta_r}\) and \(\phi_{\delta nc}^{FL} = \frac{1}{2}a_F(z_g - z_g^*) = \frac{1}{2} \cdot \frac{a_F \delta_g}{1 + \delta_y + 2\delta_y \omega_y} > 0\). In this case, the fiscal reaction parameter is unambiguously positive, while it does not depend on the monetary reaction parameter.\(^{40}\) We can establish the following result.

**Result 1:** If we allow either policy instrument to directly affect inflation, the leader authority reacts to the follower authority’s reaction parameter, hence to the follower’s preference parameter, depending on the sign of \(\delta_g \omega_i + \delta_r \omega_g\). For the fiscal leadership strategic regime, the fiscal authorities react positively, while for the regime of monetary leadership, the monetary authority reacts negatively. In cases, the leader authority might choose not to trade-off its objectives, reacting pro-cyclically.

**Proof:** For the monetary leadership strategic regime, \(\phi_{\pi}^{ML} = \phi_{\pi}^{SM} - \frac{\delta_g \omega_i (\omega_g) \phi_{\delta g}^{SM}}{a_M \delta_r}\). For \(\delta_g \omega_i + \delta_r \omega_g \neq 0\), then \(\phi_{\pi}^{ML} = \phi_{\pi}^{ML} (\phi_{g}^{SM} (a_F))\). In particular, \(\frac{\partial \phi_{\pi}^{ML}}{\partial a_F} = -\frac{\delta_g \omega_i + \delta_r \omega_g}{a_F \delta_r a_M \delta_r}\), which means that \(\text{sign} \left\{ \frac{\partial \phi_{\pi}^{ML}}{\partial a_F} \right\} = -\text{sign} \{\delta_g \omega_i + \delta_r \omega_g\}\). Moreover, for \(\delta_g \omega_i + \delta_r \omega_g > 0\), then \(\phi_{\pi}^{ML}\) can become negative. For the fiscal

\(^{39}\) See equations (A.2)-(A.3) in the Appendix A.

\(^{40}\) The case of \(\omega_g \neq 0\) and \(\omega_i = 0\) has been analyzed in Andersen (2005, 2008), for strict and flexible inflation targeting, respectively.
leadership regime, $\phi_{gnc}^{SM} = a_f \left( Z_g - \frac{1}{2} Z_i V_i \right)$. For $\omega_i \omega_g \neq 0$, then $\phi_{gnc}^{SM} = \phi_{gnc}^{SM} \left( V_1 (\phi_{\pi}^{SM} (a_M)) \right)$. In particular, $\frac{\partial \phi_{gnc}^{ML}}{\partial a_M} = \frac{1}{2} \left[ \frac{(\delta_g \omega_i + \delta_r \omega_g) \phi_{\pi}^{SM} (a_M)}{a_M \delta_r [1 + a_M (\phi_{\pi}^{SM})^2]} \right]^2$, which means that \( \text{sign} \left\{ \frac{\partial \phi_{gnc}^{ML}}{\partial a_M} \right\} = \text{sign} \{ \delta_g \omega_i + \delta_r \omega_g \}. \) Moreover, $\phi_{gnc}^{SM}$ can become negative, as $V_i > 0$.

Under monetary leadership, if $\delta_g \omega_i + \delta_r \omega_g > 0$, the monetary authority responds in a negative way to the fiscal reaction parameter, hence to the fiscal authorities’ preference parameter, which leads to a less reactionary monetary policy, i.e. $\phi_{\pi}^{ML} < \phi_{\pi}^{SM}$. In this case, fiscal (counter)-cyclicality loosens the monetary authority’s trade-off between inflation and output gap. The existence of the cost channel of monetary policy reduces the trade-off of monetary policy. The monetary reaction parameter can even become negative, which means that the monetary authority decides not to trade-off inflation with output gap. This is more so for the centralized fiscal regime, as the union-wide fiscal policy is more counter-cyclical. In the opposite case of $\delta_g \omega_i + \delta_r \omega_g < 0$, the monetary reaction parameter is unambiguously positive and larger than the corresponding one for the simultaneous-move strategic regime, as the monetary authority becomes more reactionary with the fiscal reaction parameter.

Comparing the two fiscal regimes, we get:

$$
\phi_{\pi}^{ML} - \phi_{\pi}^{ML} = \frac{(\delta_g \omega_i + \delta_r \omega_g) (\phi_{\pi}^{SM} - \phi_{gnc}^{SM})}{a_M \delta_r} = \frac{(\delta_g \omega_i + \delta_r \omega_g) a_f Z_g}{a_M \delta_r} \tag{20}
$$

If $\delta_g \omega_i + \delta_r \omega_g > 0$ (\( \delta_g \omega_i + \delta_r \omega_g < 0 \)), monetary policy is more (less) reactionary for the decentralized fiscal regime. Following equation (20), we can easily observe that \( \text{sign} \left\{ \frac{\partial (\phi_{gnc}^{ML} - \phi_{gnc}^{MC})}{\partial a_M} \right\} = \text{sign} \{ \delta_g \omega_i + \delta_r \omega_g \}. \) This means that the ratio of the authorities’ preference parameters can either increase or decrease the difference of the monetary reaction parameters for the fiscal regimes. For example, if $\delta_g \omega_i + \delta_r \omega_g > 0$, then if the weight that the monetary authority places on output-gap stabilization increases relative to the one that the fiscal authorities place, the difference between the two monetary reaction parameters decreases.

Under fiscal leadership, the national fiscal authorities react to the monetary authority’s preference parameter, as long as the two policy instruments can directly affect inflation. For $\delta_g \omega_i + \delta_r \omega_g > 0$, they respond positively, which leads to a more counter-cyclical fiscal policy. This is in contrast to the monetary leadership regime, where the monetary authority reacts negatively to the fiscal authorities’ preference parameter, hence being less reactionary. For the centralized fiscal regime, following Table 1, the union-wide fiscal policy is passive if the two policy instruments are perfect substitutes in the stabilization process.

- 24 -
In this case, the monetary authority’s response exactly offsets the impact that the union-wide fiscal stance would have on the union-wide output gap, which equals the union-wide fiscal multiplier, $\frac{\delta g}{1-\delta r}$. Moreover, the average fiscal stance is pro-cyclical under $\delta g \omega t + \delta r \omega g > 0$. Furthermore, fiscal policy for the simultaneous-move strategic regime is more counter-cyclical than under fiscal leadership, as $\phi^{FL}_{gc} = \phi^{SM}_{gc} - a_F z_i V_i \Rightarrow \phi^{FL}_{gc} < \phi^{SM}_{gc}$. Thus, the union-wide fiscal policy is more counter-cyclical under the simultaneous-move strategic regime, for both fiscal regimes.

By comparing the two fiscal regimes, we get:

$$\phi^{FL}_{gnc} - \phi^{FL}_{gc} = -a_F \left( Z_g^* - \frac{1}{2} z_i V_i \right) = \frac{\partial g_i}{\partial y_k}$$

Equation (21) shows that the difference of the two fiscal reaction parameters equals the domestic fiscal response to changes in the foreign output gap, which depends on the effect of foreign fiscal policy on domestic aggregate demand minus the monetary response. If the former (latter) effect prevails, then union-wide fiscal policy for the centralized fiscal regime would be more (less) countercyclical (or less (more) pro-cyclical). We can easily show that the union-wide fiscal policy for the decentralized fiscal regime is unambiguously more counter-cyclical when the two policy instruments are perfect substitutes in the stabilization process, whereas if there is a direct effect of fiscal policy on inflation, then the monetary preference for union-wide output-gap stabilization must exceed a critical value. In Andersen (2008), equation (21) demonstrates the horizontal coordination problem. Each fiscal authority only perceives a fraction of its fiscal decision on the common monetary policy, while the cooperative case takes into account the aggregate nature of the shock and the implied monetary response. Thus, the decentralized case delivers an inefficiency in fiscal policymaking.

The union-wide fiscal reaction parameters for the two regimes of overall policy coordination and fiscal leadership for centralized fiscal policies are very much alike, following Table 1. However, in the overall policy coordination regime the fiscal reaction parameter reacts to the monetary one in a negative way for $\delta_g \omega t + \delta_r \omega g > 0$, whereas in the fiscal leadership regime the reaction is ambiguous; in particular, it can be positive for an important monetary reaction, as $\frac{\partial \phi^{FL}_{gc}}{\partial \phi^{SM}_{\pi}} = -\frac{a_F (\delta_g \omega t + \delta_r \omega g)}{\delta r [1 + a_M (\phi^{SM}_{\pi})^2]^2} [1 - a_M (\phi^{SM}_{\pi})^2]$. In general, the case of fiscal authorities’ cooperation under fiscal leadership approximates the overall policy coordination regime for both reaction parameters. The question is now if the common central bank can in

\[41\] We can also find this result in Ferre (2008) for $\omega_g = \omega_l = 0$ and an exogenous terms-of-trade effect and in Andersen (2005, 2008) for an endogenous one, after setting $\omega_g = 0$.

\[42\] See equation A.4 in the Appendix A.
some way approximate the overall policy coordination regime under decentralized fiscal policies, i.e. how the common central bank can reduce the horizontal coordination problem. This can be the case as under fiscal leadership, in contrast with the simultaneous-move strategic regime, central bank’s preferences can affect the fiscal reaction parameter, i.e. \( a_M \) affects \( \phi_{FL} \), as long as a policy instrument can directly affect inflation.

**Corollary 1:** The monetary authority’s preferences affect the horizontal coordination problem, depending on the sign of \( \delta_g \omega_i + \delta_r \omega_g \). If it is positive (negative), the monetary authority reduces the horizontal coordination problem pursuing a more (less) flexible inflation-targeting monetary policy.

**Proof:** Following equation (20), it is straightforward that

\[
\frac{\partial (\phi_{FL} - \phi_{FL})}{\partial a_M} = -\frac{\alpha_p(\delta_g \omega_i + \delta_r \omega_g)\phi_{SM}^2}{a_M \delta_i [1 + a_M \phi_{SM}^2]^2},
\]

which means that

\[
\text{sign} \left\{ \frac{\partial (\phi_{FL} - \phi_{FL})}{\partial a_M} \right\} = -\text{sign} \{ \delta_g \omega_i + \delta_r \omega_g \}.
\]

The previous result generalizes the corresponding one by Andersen (2008) when there is a cost channel of monetary policy, too. Naturally, the cost channel of monetary policy alone unambiguously induces a more flexible inflation-targeting central banker in order to eliminate the horizontal coordination problem.

4.2 Union-wide Equilibrium Solutions

The equilibrium solutions for the union-wide macroeconomic variables (equations 15-18) require the computation of the reference parameter, \( \Omega \). Table 1 presents the reference parameter for any strategic regime for both fiscal regimes. We can make two important observations. The first one is that under perfect instrumental substitutability, the reference parameter is unambiguously positive under all strategic and fiscal regimes. In the general case where we allow the two policy instruments to directly affect inflation, then the reference parameter can become negative only under two circumstances: (i) if fiscal policy affects inflation positively, and (ii) if fiscal policy has a direct negative effect on inflation and at the same time it is more efficient in stabilizing aggregate demand (relative to inflation) than monetary policy. In general, the direct effects must be strong enough to make the reference parameter negative. The second observation is that the reference parameters for the strategic regimes of overall policy coordination and of fiscal leadership under fiscal authorities’ cooperation are unambiguously positive.
Let us now examine the case of a positive reference parameter. The union-wide inflation is positively related to demand shocks and negatively related to supply shocks, for all strategic and fiscal regimes. For the simultaneous-move strategic regime, where both the fiscal and the monetary reaction parameters are unambiguously positive, the union-wide output gap is negatively related to demand shocks and positively related to supply shocks, while the union-wide fiscal stance follows inflation. For example, a positive (negative) average supply shock that decreases (increases) average inflation leads to a reduction (increase) in both inflation and the fiscal stance at equilibrium. This means that union-wide inflation is partially stabilized, whereas the union-wide fiscal stance ends up counter-cyclical. However, the average output gap is negatively related to demand shocks and positively related to supply shocks. This means that under a positive (negative) average demand shock, the union-wide output gap decreases (increases) at equilibrium as a result of the common nominal interest rate’s reaction, hence being overly stabilized. In this case, the union-wide fiscal stance is pro-cyclical and the two policy instruments act as strategic substitutes.

We have already shown that under fiscal/monetary leadership, the leader authority’s reaction parameter might turn negative, inducing a pro-cyclical policy. Being the case, the union-wide output gap follows inflation under monetary leadership, while the union-wide fiscal stance becomes negatively (positively) related to demand (supply) shocks. In the fiscal leadership regime, the union-wide fiscal stance follows the output gap, being negatively (positively) related to demand (supply) shocks. For the two regimes of overall policy coordination and fiscal leadership for centralized fiscal policies, the union-wide fiscal stance unambiguously follows the output gap for \( \delta_g \omega_i + \delta_r \omega_g > 0 \).

**Result 2:** Under perfect instrumental substitutability, neither the strategic nor the fiscal regimes matter for pure cyclical macroeconomic stabilization at the union level; i.e., there is symbiosis of fiscal and monetary policies. However, if the policy instruments are let to directly affect inflation, then this symbiosis collapses and both the strategic and the fiscal regimes do matter at the union level.

**Proof:** It is straightforward from Table 1 that for \( \omega_g = \omega_i = 0 \), then \( \phi_\pi = \frac{\omega_r}{\alpha_M} \) and \( \Omega = \delta_r \left( 1 + \frac{\omega_r^2}{\alpha_M} \right) \) for all strategic and fiscal regimes. Thus, following equations (15) and (16) that show the equilibrium solutions for the inflation rate and the output gap at the union level, respectively, then \( \pi = -\frac{\alpha_M}{\omega_y^2 + \alpha_M} \varepsilon \) and \( y = \frac{\omega_y}{\omega_y^2 + \alpha_M} \varepsilon \).
Following the previous result, demand shocks can be fully stabilized at the union level, whereas supply shocks are only partially stabilized. Regarding the latter, the equilibrium solutions for inflation and the output gap are not equal to their long-run equilibrium values, as the authorities have (non-conflicting but) different objectives. In particular, the national fiscal authorities are concerned with fiscal stance stabilization, instead of inflation, which makes the total number of the authorities’ target variables to exceed the number of policy instruments (see, e.g., Dixit and Lambertini, 2003b; Kempf and von Thadden, 2013).

**Corollary 2:** In the general case that the two policy instruments can directly affect inflation, the choice of the particular strategic and fiscal regime becomes non-trivial for the common central bank.

*Proof:* We can compute the monetary authority’s expected loss by combining its loss function (eq. 5) with the monetary rule (eq. 13) and the union-wide equilibrium solution for inflation (eq. 15). We get:

\[
E(L_M) = \frac{1}{2} \left( 1 + \frac{\alpha M \phi \pi^2}{\Omega^2} \right) \left[ \omega_i^2 \text{Var}(u) + \delta \xi^2 \text{Var}(\varepsilon) \right]
\]

For the general case, it is straightforward from Table 1 that both the monetary reaction parameter, \( \phi \pi \), and the reference parameter, \( \Omega \), are susceptible to the particular strategic and fiscal regime. For the special case of \( \omega_g = \omega_i = 0 \), following the proof of Result 2, the monetary authority’s expected loss is exactly the same for all strategic and fiscal regimes.

**Result 3:** Under perfect instrumental substitutability, the social planner is indifferent between the two strategic regimes of simultaneous move and monetary leadership.

*Proof:* Equation (17) shows that the union-wide fiscal stance at equilibrium depends on the fiscal reaction parameter. As the latter is equal between the two strategic regimes of simultaneous move and monetary leadership, then the two regimes deliver the same union-wide fiscal stance, too. Thus, following the social planner’s loss function (eq. 9), the social planner would be indifferent between the two strategic regimes for both fiscal regimes.
4.3 Country-specific Equilibrium Solutions

We complete this section with the computation of country-specific equilibrium solutions for the decentralized fiscal regime for all the alternative strategic regimes. We solve equation (10) for both countries, to get:

\[ y_j - y = \frac{Z_e}{1+\phi_g(Z_g-Z_g^*)}(\varepsilon_j - \varepsilon_k) + \frac{1}{2} * \frac{Z_u-Z_u^*}{1+\phi_g(Z_g-Z_g^*)}(u_j - u_k), \]  

(22)

where \( Z_g - Z_g^* = \frac{\delta_g-2\delta_x\omega_g}{1+\delta_g+2\delta_x\omega_y} \) and \( Z_u - Z_u^* = \frac{1}{1+\delta_y+2\delta_x\omega_y} \). Equation (22) shows that country j’s output gap differs from the union-wide one, defined by equation (16), as long as there are asymmetric demand or supply shocks.

We proceed to the centralized fiscal regime. Following some tedious algebra, we end up with country-specific output gap for the simultaneous-move (and the monetary leadership) regime, as:

\[ y_j = \frac{1+a_F Z_g(Z_g-Z_g^*)}{1+a_F(Z_g-Z_g^*)^2} y + \frac{Z_e}{1+a_F(Z_g-Z_g^*)^2}(\varepsilon_j - \varepsilon_k) + \frac{1}{2} * \frac{Z_u-Z_u^*}{1+a_F(Z_g-Z_g^*)^2}(u_j - u_k) \]  

(23)

Now, the country-specific output gap differs from the union-wide one even under common shocks. We can easily show that the nominator is higher than the denominator, which means that a change in the union-wide output gap leads to a bigger change to the country-specific one. Moreover, the nominator for the union-wide output gap is the same with the denominator for shocks’ asymmetries in the decentralized case. Thus, under centralized fiscal policies, the country-specific output gap is more volatile to shocks’ asymmetries than under decentralized fiscal policies.

We complete this section with the fiscal leadership strategic regime. For the decentralized case, we use equation (22). The focus is on the determination of the sign of the denominator, meaning 1 + \( a_F Z_g(Z_g-Z_g^*) \), following the fiscal reaction parameter from Table 1. If it is positive (negative), then under demand or supply shocks’ asymmetries country j’s output gap will be higher (lower) than the union-wide one, whereas the other country k’s will be lower (higher). For centralized fiscal policies, following again some tedious algebra, we end up with:

\[ y_j - y = \frac{Z_e}{1+a_F(Z_g-Z_g^*)^2}(\varepsilon_j - \varepsilon_k) + \frac{1}{2} * \frac{Z_u-Z_u^*}{1+a_F(Z_g-Z_g^*)^2}(u_j - u_k), \]  

(24)

---

43 We subtract the two symmetric equations for the two countries and we incorporate the country-specific fiscal rules defined by equation (11).

44 The country-specific equilibrium solution for the overall policy coordination regime can be found in the Appendix B.
where the sign of the denominator is unambiguous. Thus, under positive demand or supply shocks’ asymmetries, country j’s output gap is higher than the union-wide one. A simple comparison with the simultaneous-move (monetary leadership) strategic regime (eq. 23) demonstrates that under idiosyncratic shocks all the regimes deliver identical results. On the contrary, under common shocks, the country-specific output gap for the fiscal leadership regime equals the union-wide one, while the corresponding one from the simultaneous-move (monetary leadership) regime is higher than the union-wide one. We can establish the following result.

**Result 4:** Let perfect instrumental substitutability and common shocks. For demand shocks, there is overall symbiosis between monetary and fiscal policies; i.e., all strategic and fiscal regimes deliver the same equilibrium solutions for both union-wide and country-specific inflation and output gap, which are also equal to the target (long-run equilibrium) values. For supply shocks, there is overall symbiosis only for the decentralized fiscal regime, while the equilibrium solutions depart from the target values. For the fiscal leadership regime, the centralized fiscal regime delivers the same equilibrium solutions, too.

**Proof:** Following the proof of Result 2, we only need to show that \( y_j = y \) for common shocks. For the decentralized fiscal regime, it is straightforward from equation (22), while for the centralized case under fiscal leadership, it is straightforward from equation (24). Following equation (23) for the simultaneous-move and the monetary leadership regimes, for demand shocks, \( y = 0 \), which means that \( y_j = y \). For supply shocks, \( y_j \neq y \), as \( y = \frac{a_M}{a_S + a_M} \varepsilon \), hence indifferent to zero. ■

5. A Welfare Analysis of the Alternative Fiscal Regimes

This section is dedicated to a welfare analysis for the monetary authority, the national fiscal authorities and the social planner, regarding the two alternative fiscal regimes. We begin with the monetary authority.

5.1 The Monetary Authority

Following Corollary 2, we know that there is a preferable strategic regime for the monetary authority when the two policy instruments can directly affect inflation. We can establish the following propositions.\(^{45}\)

\(^{45}\) We compare the variances of union-wide inflation and the output gap for the two alternative fiscal regimes, following the equilibrium solutions (equations 15 and 16). See Appendix C.1.
**Proposition 1:** For the simultaneous-move strategic regime, the monetary authority prefers the decentralized fiscal regime if and only if $0 < \delta_g \omega_i + \delta_r \omega_g < \frac{2\delta_r [1 + a_M (\phi_{cM}^S)^2]}{\phi_{cM}^S a_F (2Z_g + Z_g^*)}$. Otherwise, it prefers the centralized fiscal regime.

**Proof:** Following equations (C.1) and (C.2) in Appendix C.1, we get $\Omega_{SM}^c - \Omega_{SM}^{nc} = - (\delta_g \omega_i + \delta_r \omega_g) \phi_{cM}^S a_F Z_g^*$. and $\Omega_{SM}^c + \Omega_{SM}^{nc} = 2 \delta_r [1 + a_M (\phi_{cM}^S)^2] - (\delta_g \omega_i + \delta_r \omega_g) \phi_{cM}^S a_F (2Z_g + Z_g^*)$. For $\delta_g \omega_i + \delta_r \omega_g < 0$, then $\Omega_{SM}^c - \Omega_{SM}^{nc} = |\delta_g \omega_i + \delta_r \omega_g| \phi_{cM}^S a_F (2Z_g + Z_g^*) > 0$ and $\Omega_{SM}^c + \Omega_{SM}^{nc} = 2 \delta_r [1 + a_M (\phi_{cM}^S)^2] + |\delta_g \omega_i + \delta_r \omega_g| \phi_{cM}^S a_F (2Z_g + Z_g^*) > 0$. Thus, $Var(\pi)^{nc}_{SM} > Var(\pi)^c_{SM}$ and $Var(y)^{nc}_{SM} > Var(y)^c_{SM}$, which also leads to $E(LM)^{nc}_{SM} > E(LM)^c_{SM}$, following equation (C.4). In the general case of $\omega_i$ and $\omega_g$ positive, which means that $\delta_g \omega_i + \delta_r \omega_g > 0$, then for $E(LM)^{nc}_{SM} < E(LM)^c_{SM}$ must be $\Omega_{SM}^c + \Omega_{SM}^{nc} > 0 \Rightarrow \delta_g \omega_i + \delta_r \omega_g < \frac{2\delta_r [1 + a_M (\phi_{cM}^S)^2]}{\phi_{cM}^S a_F (2Z_g + Z_g^*)}$, as $\Omega_{SM}^c - \Omega_{SM}^{nc} < 0$. Thus, $0 < \delta_g \omega_i + \delta_r \omega_g < \frac{2\delta_r [1 + a_M (\phi_{cM}^S)^2]}{\phi_{cM}^S a_F (2Z_g + Z_g^*)}$.

Proposition 1 shows that the answer to the question which fiscal regime is preferable for the monetary authority depends on the specific structural combination of parameters, $\delta_g \omega_i + \delta_r \omega_g$. In particular, the monetary authority prefers fiscal authorities’ cooperation if (i) fiscal policy directly affects inflation in a negative way and at the same time fiscal policy is less efficient in stabilizing aggregate demand than monetary policy, and (ii) if the two policy instruments directly affect positively the inflation rate and $\delta_g \omega_i + \delta_r \omega_g$ is high enough. In the case of $\delta_g \omega_i + \delta_r \omega_g > 0$, the fraction $\frac{2\delta_r [1 + a_M (\phi_{cM}^S)^2]}{\phi_{cM}^S a_F (2Z_g + Z_g^*)}$ defines a critical value over which the monetary authority prefers the centralized fiscal regime. This critical value is positively related to the weight that the common central bank places on output-gap stabilization, $a_M$, whereas it is negatively related to the corresponding one for the national fiscal authorities. The former observation means that a more flexible inflation-targeting central banker increases this critical value and eases the case where the monetary authority would prefer the decentralized case; so, there is no need to promote fiscal authorities’ cooperation. On the contrary, if the common central bank follows a less flexible inflation-targeting approach, the critical value in favor of the decentralized case would be low, which makes $\delta_g \omega_i + \delta_r \omega_g$ have more chances to exceed this critical value, hence being in favor of the centralized case. Finally, if the national fiscal authorities care a lot about country-specific output-gap stabilization, it is more possible that the monetary authority would prefer the centralized fiscal regime.
**Proposition 2:** For the fiscal leadership regime, in the special case of \((\delta_g \omega_i + \delta_r \omega_g) \left( Z_g^* - \frac{1}{2} Z_i V_i \right) > 0\), both union-wide inflation and the output gap are less volatile for the decentralized fiscal regime, which makes this regime preferable.

**Proof:** Following equations (C.1) and (C.2) in the Appendix C.1, we get

\[
\Omega^c_{FL} - \Omega^nc_{FL} = -(\delta_g \omega_i + \delta_r \omega_g) \phi^SM_a F \left( Z_g^* - \frac{1}{2} Z_i V_i \right) \]

and

\[
\Omega^c_{FL} + \Omega^nc_{FL} = 2\Omega^c_{FL} + (\delta_g \omega_i + \delta_r \omega_g) \phi^SM_a F \left( Z_g^* - \frac{1}{2} Z_i V_i \right),
\]

where \(\Omega^c_{FL} > 0\). Thus, if \((\delta_g \omega_i + \delta_r \omega_g) \left( Z_g^* - \frac{1}{2} Z_i V_i \right) > 0\), then \(\Omega^c_{FL} - \Omega^{nc}_{FL} < 0\) and \(\Omega^c_{FL} + \Omega^{nc}_{FL} > 0\), which lead to \(Var(\pi)^{nc}_{FL} < Var(\pi)^{c}_{FL}\) and \(Var(y)^{nc}_{FL} < Var(y)^{c}_{FL}\), and \(E(LM)^{nc}_{FL} < E(LM)^{c}_{FL}\), following equation (C.4).

Following Proposition 2, if both policy instruments directly affect inflation in a positive manner, meaning \(\delta_g \omega_i + \delta_r \omega_g > 0\), then the monetary authority prefers the decentralized case if and only if the country-specific fiscal policy responds counter-cyclically to changes in foreign aggregate demand, as \(\frac{\partial g_j}{\partial y_k} = -a_F \left( Z_g^* - \frac{1}{2} Z_i V_i \right)\) following equation (21).

5.2 The Two National Fiscal Authorities

In this subsection, the variances of the country-specific output gap between the two alternative fiscal regimes are compared, as well as the expected losses of both national fiscal authorities, in order to determine the preferable fiscal regime. Only common and idiosyncratic (perfectly asymmetric) demand and supply shocks are considered, while it is assumed that shocks have the same variances, following Beetsma and Jensen (2005).\(^{46}\) We will see that results are shock-independent.

**Result 5:** The national fiscal authorities prefer to coordinate their policies for all strategic regimes under idiosyncratic shocks, even when the two policy instruments can directly affect inflation.

\(^{46}\) In particular, we assume that \(\sigma^2_{\epsilon j} = \sigma^2_{\epsilon i} = \sigma^2_{\epsilon}\) for supply shocks and \(\sigma^2_{u j} = \sigma^2_{u i} = \sigma^2_{u}\) for demand shocks, which simply means that shocks are of equal size. We may justify this assumption as countries are identical and of the same size.
Proof: We use equation (22) for the case of decentralized fiscal policies and equations (23) and (24) for the centralized case, for the simultaneous-move and the fiscal leadership strategic regimes, respectively, together with the fiscal rules and the expected losses. Starting with the former regime, we get:

\[
E \left( L_{Fj} \right)_{SM}^{nc} - E \left( L_{Fj} \right)_{SM}^{c} = \frac{1}{2} \ast \frac{\left( a_FZ_g \right)^2 \left( Z_u - Z_u^* \right)^2 \sigma_u^2 + 4Z_u \sigma_u^2}{\left[ 1 + a_FZ_g (Z_g - Z_g^*) \right] \left[ 1 + a_F (Z_g - Z_g^*) \right]^2} > 0
\]

For the fiscal leadership regime, we get:

\[
E \left( L_{Fj} \right)_{FL}^{nc} - E \left( L_{Fj} \right)_{FL}^{c} = \frac{1}{2} \ast \frac{a_F^2 \left( Z_g^* - Z_g^* \right)^2 \left( Z_u - Z_u^* \right)^2 \sigma_g^2 + 4Z_g \sigma_g^2}{\left[ 1 + \phi_{nc}^F (Z_g - Z_g^*) \right] \left[ 1 + a_F (Z_g - Z_g^*) \right]^2} > 0.
\]

For the monetary leadership strategic regime, it turns out that the results are exactly the same with the simultaneous-move one. This is so, as under idiosyncratic shocks, the monetary reaction parameter cannot affect country-specific macroeconomic variables and the fiscal reaction parameters for the case of monetary leadership equal the corresponding ones from the simultaneous-move case. Result 5 embraces the traditional Optimum Currency Area (OCA) theory that asymmetric shocks increase the costs of being in a monetary union. Thus, fiscal authorities’ cooperation can improve the functioning of a monetary union (Foresti, 2017). It also extends Della Posta and De Bonis (2009) for the case where the two policy instruments can directly affect inflation, including also the monetary leadership strategic regime. We proceed the analysis with common shocks.

Proposition 3: In the special case where the two policy instruments are perfect substitutes in the stabilization process, the national fiscal authorities would unambiguously prefer not to coordinate their fiscal policies when their economies are hit by common shocks under the simultaneous-move strategic regime, while under fiscal leadership they prefer the decentralized fiscal regime if and only if country-specific fiscal policies respond in the same way to both domestic and foreign changes in aggregate demand, meaning either counter-cyclically or pro-cyclically.

Proof: See Appendix C.2.

The previous result is consistent with Della Posta and De Bonis (2009). It follows the rationale of traditional OCA theory that fiscal policies’ coordination is most likely to be undesirable when shocks between member-states are highly correlated (Foresti, 2017). It also reinforces Rogoff’s result that partial
cooperation between (some) players may be counterproductive (see Rogoff, 1985b). The difference between the two strategic regimes is attributed to the fact that the two national fiscal authorities under simultaneous move do not take into account the monetary response to their fiscal actions. However, even in the case where the two policy instruments are perfect substitutes, fiscal policies’ coordination can become welfare-improving under fiscal leadership, challenging earlier findings (see, e.g., Beetsma and Bovenberg, 1998). In the next sub-section, we investigate this result at the union level, i.e. from the social planner’s perspective.

5.3 The Union-Wide Fiscal Stance and Social Planner’s Preferences

This sub-section examines if the social planner would prefer the national fiscal authorities to coordinate their policies under the simultaneous-move strategic regime. However, we begin by comparing the variances of the union-wide fiscal stance. We establish the following propositions.

**Proposition 4:** The decentralized fiscal regime delivers a less volatile union-wide fiscal stance under
\[ \delta_g \omega_i + \delta_r \omega_g < \frac{(2Z_g + Z_g^*)\delta_r [1 + a_M(\phi^{SM}_\pi)^2]}{2a_FZ_g(Z_g + Z_g^*)\phi^{SM}_\pi}. \]

Both a more flexible inflation-targeting central banker and a more fiscal-policy concerned fiscal authority work in favor of the decentralized case.

**Proof:** By subtracting the two variances for the union-wide fiscal stance, we get:
\[
V\text{ar}(g)^{nc}_{SM} - V\text{ar}(g)^c_{SM} = \left(\phi^{SM}_{\pi}\right)^2 \omega^2 V\text{ar}(\omega) + \delta_r^2 V\text{ar}(\varepsilon) \frac{(\phi^{IR}_{gnc} \Omega^c_{SM} - \phi^{IR}_{gc} \Omega^{nc}_{SM})(\phi^{IR}_{gnc} \Omega^c_{SM} + \phi^{IR}_{gc} \Omega^{nc}_{SM})}{(\Omega^c_{SM} \Omega^{nc}_{SM})^2},
\]

where \( \phi^{SM}_{gnc} \Omega^c_{SM} - \phi^{SM}_{gc} \Omega^{nc}_{SM} = -a_FZ_g\delta_r [1 + a_M(\phi^{SM}_\pi)^2] < 0 \) and \( \phi^{SM}_{gnc} \Omega^c_{SM} + \phi^{SM}_{gc} \Omega^{nc}_{SM} = a_F(2Z_g + Z_g^*)\delta_r [1 + a_M(\phi^{SM}_\pi)^2] - 2a_FZ_g(Z_g + Z_g^*)(\delta_g \omega_i + \delta_r \omega_g)\phi^{SM}_\pi \). Thus, \( V\text{ar}(g)^{nc}_{SM} < V\text{ar}(g)^c_{SM} \) if and only if \( \delta_g \omega_i + \delta_r \omega_g < \frac{(2Z_g + Z_g^*)\delta_r [1 + a_M(\phi^{SM}_\pi)^2]}{2a_FZ_g(Z_g + Z_g^*)\phi^{SM}_\pi} \). It is straightforward that this fraction is positively related to the weight that the central bank places on output-gap stabilization and negatively related to the corresponding one for the national fiscal authorities.

**Proposition 5:** Fiscal authorities’ cooperation can become welfare-improving under the simultaneous-move strategic regime when the two policy instruments can directly affect inflation.

**Proof:** See Appendix C.3.
For the special case of perfect instrumental substitutability, the social planner prefers the national fiscal authorities not to cooperate with one another. This result is consistent with Rogoff’s (1985b) classical result that partial cooperation among (some) players is not welfare improving, whereas it is also consistent with Beetsma and Bovenberg (1998). However, for the general case of \( \delta_g \omega_i + \delta_r \omega_g \neq 0 \) we cannot get a clear-cut result, which means that fiscal authorities’ cooperation after all can become welfare improving when the two policy instruments can directly affect inflation, contradicting with the previous authors.

6. Comparison of the Alternative Strategic Regimes

This section is dedicated to a comparison of the alternative strategic regimes regarding union-wide (cyclical) macroeconomic stabilization. In particular, we compare the variances of our main macroeconomic variables, namely the output gap, inflation and the fiscal stance for the overall policy coordination regime, the simultaneous-move one and the regime of fiscal leadership. We also consider both fiscal regimes. We will see that the comparison mainly depends on a specific configuration of structural parameters that shows which policy instrument is more effective in stabilizing aggregate demand relative to inflation, namely \( \delta_g \omega_i + \delta_r \omega_g \).

6.1 The Simultaneous-Move Regime versus the Fiscal Leadership Regime

The comparisons of the variances of the main macroeconomic variables are presented in the Appendix D.1. Our main results are as follows.

**Proposition 6:** Under perfect instrumental substitutability, the simultaneous-move strategic regime is inferior to the fiscal leadership one under supply shocks, for both fiscal regimes (see, e.g., Dixit and Lambertini, 2001). However, when we allow for the two policy instruments to directly affect inflation, the simultaneous move can become a superior strategic regime, independent on shocks.

**Proof:** See Appendix D.2.

**Corollary 3:** The common central bank unambiguously prefers the simultaneous-move strategic regime over the fiscal leadership one under fiscal policies’ coordination, when \( \delta_g \omega_i + \delta_r \omega_g < 0 \).
Proof: In the Appendix D.1, we show that $\text{Var}(\pi)^c_{SM} < \text{Var}(\pi)^c_{FL}$ and $\text{Var}(y)^c_{SM} < \text{Var}(y)^c_{FL}$ for $\delta_{i}\omega_{i} + \delta_{f}\omega_{g} < 0$. Following the monetary authority’s expected loss, it is straightforward that $E(L_M)^c_{SM} < E(L_M)^c_{FL}$. The result is shock-independent.

Corollary 3 shows that the simultaneous-move strategic regime can become superior to the fiscal leadership strategic regime for pure cyclical macroeconomic stabilization at the union level if the national fiscal authorities coordinate their policies. In fact, it shows that if fiscal policy can directly affect inflation in a negative manner and fiscal policy is less efficient on aggregate demand than monetary policy, then this is unambiguously so. This result contradicts with the famous result of Dixit and Lambertini (2001) for the superiority of the leadership regimes over the simultaneous-move one.

6.2 The Overall Policy Coordination Regime versus the Fiscal Leadership for Coordinated Fiscal Policies

We have seen from our analysis so far that the two regimes are alike. For the special case of perfect substitutability, we can determine three specific cases that refer to preferences where the fiscal leadership regime for coordinated fiscal policies becomes superior.

Proposition 7: For the special case of perfect instrumental substitutability, the fiscal leadership regime under fiscal authorities’ cooperation becomes unambiguously welfare-superior under three specific circumstances regarding the agents’ (monetary authority, fiscal authorities, social planner) preference on output-gap stabilization: (i) when all the authorities have the same preference, (ii) when the monetary and the fiscal authorities jointly share society’s preferences and fiscal authorities’ preference is less than double (as the fiscal authorities are two) from the monetary authority’s preference, and (ii) when the monetary authority has society’s preference and at the same time the national fiscal authorities’ preference is lower.

Proof: See Appendix D.3.

7. Conclusion

We examine the policy mix in a monetary union in a strategic context when the two policy instruments can directly affect inflation. In particular, we use a reduced-form two-country monetary union model based
on the New Keynesian framework, in order to study the strategic interactions of the monetary authority and the national fiscal authorities under all strategic regimes and fiscal authorities’ cooperation. We find that the cost channel of monetary policy limits union-wide demand shocks’ stabilization at the union level, that the leader policy authority might choose not to trade-off its objectives, acting pro-cyclically, that fiscal policy becomes non-neutral at the union level, and that fiscal authorities’ cooperation may end up welfare-enhancing.

The most important result from our macroeconomic and welfare comparisons is that when we allow the two policy instruments to directly affect inflation, then the choice of both the fiscal and the institutional strategic regimes does matter at the union level, even when all the authorities have non-conflicting (but different) objectives. Thus, the irrelevance conditions of Kempf and von Thadden (2013) and the symbiosis result of Dixit and Lambertini (2001, 2003a) do not hold anymore. In particular, the monetary authority becomes interested in the specific fiscal and strategic regimes. For the simultaneous-move strategic regime, we succeed in defining the preferable fiscal regime for the monetary authority, depending on the combination of parameters that show which policy instrument is more efficient in stabilizing aggregate demand relative to inflation. We also find that the simultaneous-move strategic regime can become welfare-improving. In particular, if fiscal policy can directly affect inflation negatively and at the same time fiscal policy is less efficient in stabilizing aggregate demand (relative to inflation) than monetary policy, then the common central bank prefers the simultaneous-move regime under fiscal authorities’ cooperation, as it delivers less volatile union-wide inflation and output gap. Both results are of particular importance, as in cases of extreme shocks and events, like the Great Recession, the simultaneous-move strategic regime might be the most appropriate to analyse fiscal/monetary policy interactions (Dai and Sidiropoulos, 2011). Empirical evidence provided by Hughes Hallett (2005) are also in favour of the simultaneous-move strategic regime for the EMU.

In the special case that the two policy instruments are perfect substitutes in the stabilization process, we have shown that (i) the alternative fiscal regimes do not matter, (ii) the simultaneous-move strategic regime delivers exactly the same results with the monetary leadership regime at the union level for all macroeconomic variables, which further means that both the common central bank and the social planner are indifferent between those two strategic regimes, and (iii) that the simultaneous-move strategic regime coincide with the fiscal leadership for inflation and the output gap, which makes the common central bank indifferent between those two strategic regimes, but the union-wide fiscal stance is less volatile under fiscal leadership, which makes this particular regime desirable for the social planner. We further show that for common demand shocks, there is overall symbiosis between fiscal and monetary policies; i.e., all strategic and fiscal regimes deliver the same equilibrium solutions for both union-wide and country-specific inflation
and output gap, which are also equal to the target (long-run equilibrium) values. For common supply shocks, there is overall symbiosis only for the decentralized fiscal regime, but equilibrium solutions depart from the target values. This results from the fiscal stance being both an instrument and a target for the national fiscal authorities (see Dixit and Lambertini, 2003a; Kempf and von Thadden, 2013). For the fiscal leadership regime, the centralized fiscal regime delivers the same equilibrium solutions, too.

The similarity of the fiscal leadership regime under fiscal authorities’ cooperation with the overall policy coordination regime means that the former regime enhances overall policy coordination (see, e.g., Andersen, 2008; Hughes Hallett and Weymark, 2007). In the special case of perfect instrumental substitutability, where the two policy instruments are equally efficient, the union-wide fiscal stance is equally volatile for both strategic regimes. Moreover, in this case, the fiscal leadership regime under centralized fiscal policies delivers a less volatile union-wide inflation. As far as the social planner is concerned, the fiscal leadership regime under fiscal authorities’ cooperation becomes unambiguously welfare superior under three specific circumstances regarding the agents’ (monetary authority, fiscal authorities, social planner) preference on output-gap stabilization: (i) when the monetary and the fiscal authorities share the same preference, which also equals to the social planner’s preference, (ii) when the monetary and the fiscal authorities jointly share society’s preferences and fiscal authorities’ preference is less than double (as the fiscal authorities are two) of the monetary authority’s preference, and (ii) when the monetary authority has the social planner’s preference and at the same time the national fiscal authorities’ preference is lower. In the second case, the overall policy coordination regime is desirable when fiscal authorities’ preference is larger than double than the monetary authority’s preference, while the two regimes are identical if the fiscal authorities’ preference is equal to the double of the monetary authority’s preference.
<table>
<thead>
<tr>
<th>Reaction &amp; Reference Parameters</th>
<th>Overall Policy Coordination Regime</th>
<th>Simultaneous-Move Regime</th>
<th>Fiscal Leadership Regime</th>
<th>Monetary Leadership Regime</th>
</tr>
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<tbody>
<tr>
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<tr>
<td><strong>$\Phi_\pi$</strong></td>
<td>$\frac{a_y \delta_r - (1 - \delta_y) \omega_i}{(a_F + a_M) \delta_r}$</td>
<td>$\frac{\omega_r \delta_r - (1 - \delta_y) \omega_i}{a_M \delta_r}$</td>
<td>$\frac{\omega_r \delta_r}{a_M \delta_r}$</td>
<td>$\frac{\delta_y \omega_i + \delta_r \omega_y + \delta_r \omega_y}{a_M \delta_r}$</td>
</tr>
<tr>
<td><strong>$\Phi_g$</strong></td>
<td>$\frac{\delta_y \omega_i + \delta_r \omega_y}{\delta_r \phi_{\pi}}$</td>
<td>$a_y Z_g$</td>
<td>$a_F (Z_g + \delta_r \omega_i)$</td>
<td>$a_F (Z_g + \delta_r \omega_i)$</td>
</tr>
<tr>
<td><strong>$\Omega$</strong></td>
<td>$\delta_t \left[ 1 + (a_F + a_M)(\phi_{\pi}^{SM})^2 \right]$</td>
<td>$- \delta_t Z_g (\delta_y \omega_i + \delta_r \omega_y)$</td>
<td>$- \delta_t Z_g (\delta_y \omega_i + \delta_r \omega_y)$</td>
<td>$- \delta_t Z_g (\delta_y \omega_i + \delta_r \omega_y)$</td>
</tr>
</tbody>
</table>

‘OC’ stands for Overall (policy) Coordination, ‘SM’ for Simultaneous Move and ‘ML’ for Monetary Leadership; $V_t = \frac{\delta_g (1 + \omega_y \phi_{\pi}^{SM} + (1 - \delta_y) \omega_y \phi_{\pi}^{SM})}{\delta_t [1 + a_M (\phi_{\pi}^{SM})^2]}$. 

| Table 1 |
|---------|---------|---------|---------|---------|
| Reaction & Reference Parameters | Overall Policy Coordination Regime | Simultaneous-Move Regime | Fiscal Leadership Regime | Monetary Leadership Regime |
| No Cooperation | Cooperation | No Cooperation | Cooperation | No Cooperation | Cooperation |
| $\phi_{\pi}$ | $\frac{a_y \delta_r - (1 - \delta_y) \omega_i}{(a_F + a_M) \delta_r}$ | $\frac{\omega_r \delta_r - (1 - \delta_y) \omega_i}{a_M \delta_r}$ | $\frac{\omega_r \delta_r}{a_M \delta_r}$ | $\frac{\delta_y \omega_i + \delta_r \omega_y + \delta_r \omega_y}{a_M \delta_r}$ |
| $\phi_g$ | $\frac{\delta_y \omega_i + \delta_r \omega_y}{\delta_r \phi_{\pi}}$ | $a_y Z_g$ | $a_F (Z_g + \delta_r \omega_i)$ | $a_F (Z_g + \delta_r \omega_i)$ |
| $\Omega$ | $\delta_t \left[ 1 + (a_F + a_M)(\phi_{\pi}^{SM})^2 \right]$ | $- \delta_t Z_g (\delta_y \omega_i + \delta_r \omega_y)$ | $- \delta_t Z_g (\delta_y \omega_i + \delta_r \omega_y)$ | $- \delta_t Z_g (\delta_y \omega_i + \delta_r \omega_y)$ |
Appendix

A. Model Solution – The Construction of Table 1

The overall policy coordination regime requires the minimization of equation (8) with respect to the common nominal interest rate and the two country-specific fiscal stances, subject to the country-specific aggregate demand equations (10) and the union-wide aggregate demand and the PC equations, namely equations (3) and (4), respectively. For the simultaneous-move strategic regime, the monetary authority minimizes equation (5) with respect to the common nominal interest rate, subject to the union-wide non-policy block of equations, namely equations (3) and (4). The two national fiscal authorities minimize equation (6) with respect to country-specific fiscal stance, subject to country-specific aggregate demand (eq. 10). For the centralized case, the two national fiscal authorities minimize equation (7) with respect to both the country-specific fiscal stances, subject to both the country-specific aggregate demand equations (10). In particular, the country-specific fiscal rule can be computed as:

\[ g_j = -a_F (Z_g y_j + Z_g^* y_k) \]  

(A.1)

By averaging equation (A.1) with the corresponding one for country \( k \), we end up with the (union-wide) fiscal reaction parameter for the case of centralized fiscal policies, which is shown in Table 1. For the two leadership regimes, the leader authority incorporates in its decision-making problem the follower authority’s policy rule. In particular, under fiscal leadership, the two national fiscal authorities solves their optimization program also subject to the monetary rule (eq. 11), while under monetary leadership the common central bank solves its own program subject to the union-wide fiscal rule (eq. 12).

For the fiscal leadership strategic regime, we first solve the monetary authority’s problem to derive the monetary rule and the common nominal interest rate with respect to the average fiscal stance. Naturally, both are equal to the corresponding ones for the simultaneous-move strategic regime. Combining the monetary rule with the descriptive equations for the monetary union, meaning equations (3) and (4), we can compute the common nominal interest rate as a function of the union-wide demand and supply shocks, and the union-wide fiscal stance. Thus:

\[ i = \frac{\delta_g (1 + \omega_y \phi^S M) + (1 - \delta_y) \omega_y \phi^S M}{\delta_g [1 + a_M (\phi^S M)^2]} g + \frac{1 + \omega_y \phi^S M}{\delta_g [1 + a_M (\phi^S M)^2]} u - \frac{(1 - \delta_y) \phi^S M}{\delta_g [1 + a_M (\phi^S M)^2]} \epsilon \]  

(A.2)

Equation (A.2) represents the monetary authority’s reaction function, which works as a constraint to the national fiscal authorities. For fiscal authorities’ cooperation, the country-specific fiscal rule is defined as:

\[ g_j = -a_F (Z_g y_j + Z_g^* y_k - Z_i V_i y) \]  

(A.3)
Equation (A.3) shows that each fiscal authority addresses its own output gap as before, as \( \frac{\partial g_j}{\partial y_j} = -a_F \left( Z_g - \frac{1}{2} Z_i V_i \right) \), but it also addresses changes in the foreign country’s output gap, following \( \frac{\partial g_j}{\partial y_k} = -a_F \left( Z_g^* - \frac{1}{2} Z_i V_i \right) \).

Following Table 1, we can compare the two fiscal reaction parameters for the two fiscal regimes. We get:

\[
\phi_{gnc}^{FL} - \phi_{gc}^{FL} = \frac{a_F \left[ \delta_r - 2 \delta_t \omega_g + \frac{\phi_{SM}^{PL}}{\delta_r} \left[ \delta_r \left( \omega_y \delta_g + \left( 1 + \delta_y \right) \omega_g \right) + 2 \omega_t \left( \delta_t \left( \omega_y \delta_g + \omega_g \right) + \delta_y \left( \delta_g - \delta_t \omega_g \right) \right) \right] \right]}{2 \left[ 1 + \delta_y + 2 \delta_t \omega_y \right] \left[ 1 + a_M \left( \phi_{SM}^{FL} \right)^2 \right]}
\]  
(A.4)

If we set \( \omega_g = \omega_i = 0 \), we get \( \phi_{gnc}^{FL} - \phi_{gc}^{FL} = \frac{a_F \delta_r - \delta_t \omega_g + \frac{\phi_{SM}^{PL}}{\delta_r} \left[ \omega_y \delta_g + \left( 1 + \delta_y \right) \omega_g \right] \left[ 1 + a_M \left( \phi_{SM}^{FL} \right)^2 \right]}{2 \left[ 1 + \delta_y + 2 \delta_t \omega_y \right]} > 0 \). Under \( \omega_i = 0 \) and \( \omega_g \neq 0 \), we get \( \phi_{gnc}^{FL} - \phi_{gc}^{FL} = \frac{a_F \left[ \delta_r - 2 \delta_t \omega_g + \frac{\phi_{SM}^{PL}}{\delta_r} \left[ \omega_y \delta_g + \left( 1 + \delta_y \right) \omega_g \right] \right]}{2 \left[ 1 + \delta_y + 2 \delta_t \omega_y \right] \left[ 1 + a_M \left( \phi_{SM}^{FL} \right)^2 \right]} \), where \( \phi_{SM} = \frac{\omega_y}{a_M} \). Thus, for \( \phi_{gnc}^{FL} - \phi_{gc}^{FL} > 0 \), we need the nominator to be positive, which holds for \( a_M > -\frac{\omega_y \delta_g + \left( 1 + \delta_y \right) \omega_g}{\delta_r - 2 \delta_t \omega_g} \). Under \( \omega_g > 0 \), the case where \( \omega_g < \frac{\delta_g}{2\delta_r} \) delivers a negative value, which simply means that it holds all times, as \( a_M \) cannot take negative values. The same holds for \( \omega_y \delta_g + \left( 1 + \delta_y \right) \omega_g > 0 \) under \( \omega_g < 0 \).

**B. The Country-specific Solution for the Overall Policy Coordination Regime**

We provide the country-specific equilibrium solution for the fiscal-monetary (overall) policy coordination regime. The country-specific fiscal rule is given by:

\[
g^{OC}_j = \left[ \frac{\omega_y (Z_g + Z_g^*) + \omega_g}{\phi_{SM}^{OC}} - a_M (Z_g + Z_g^*) - 2 a_F Z_g^* \right] y^{OC} - a_F (Z_g - Z_g^*) y^{OC}_j
\]  
(B.1)

In order to compute the country-specific output-gap at equilibrium, we subtract the two aggregate demand equations (10), in order to extract the country-specific fiscal stance, and then we incorporate it, together with the union-wide fiscal and monetary rules, to the country-specific fiscal rule (eq. B.1). After some tedious algebra, we end up with:

\[
y^{OC}_j = \frac{1}{1 + a_F (Z_g - Z_g^*)^2} \left\{ 1 + \phi^{OC}_g (Z_g - Z_g^*) \left[ 1 + \frac{\delta_r \omega_y (Z_g + Z_g^*) + \omega_g}{\delta_g \omega_i + \delta_r \omega_g} \right] - a_M (Z_g + Z_g^*) + 2 a_F Z_g^* \} y^{OC} + \frac{z_e}{1 + a_F (Z_g - Z_g^*)^2} \left( \epsilon_j - \epsilon_k \right) + \frac{1}{2} \frac{z_u - z_0^*}{1 + a_F (Z_g - Z_g^*)^2} (u_j - u_k)
\]  
(B.2)
At equilibrium, we incorporate the solution for the union-wide output gap following equation (16).

C. A Welfare Analysis of the Alternative Fiscal Regimes (Section 5)

C.1 The Monetary Authority’s Welfare Analysis (Section 5.1)

We compare the variances for union-wide inflation and the output gap. We get:

\[
V\var(\pi)_{IR}^{nc} - V\var(\pi)_{IR}^c = \frac{\omega_i^2 V\var(u) + \delta_i^2 V\var(\epsilon)}{(\omega_i^2 + \delta_i^2)} (\Omega_{IR}^{nc} - \Omega_{IR}^c) (\Omega_{IR}^{nc} + \Omega_{IR}^c) \tag{C.1}
\]

\[
V\var(y)_{IR}^{nc} - V\var(y)_{IR}^c = (\phi_{II}^R)^2 [V\var(\pi)_{IR}^{nc} - V\var(\pi)_{IR}^c], \tag{C.2}
\]

where ‘IR’ stands for ‘Institutional Regimes’. The former equation holds for all strategic regimes, while the latter holds only for the simultaneous move and the fiscal leadership regimes. For the monetary leadership regime, it becomes:

\[
V\var(y)_{ML}^{nc} - V\var(y)_{ML}^c = \frac{\omega_i^2 V\var(u) + \delta_i^2 V\var(\epsilon)}{(\omega_i^2 + \delta_i^2)} (\phi_{II}^{ML} \Omega_{ML}^{nc} - \phi_{II}^{ML} \Omega_{ML}^c) (\phi_{II}^{ML} \Omega_{ML}^{nc} + \phi_{II}^{ML} \Omega_{ML}^c) \tag{C.3}
\]

We can compare the monetary authority’s loss under the two alternative fiscal regimes for the simultaneous move and the fiscal leadership regime. It is straightforward that:

\[
E(L_M)_{IR}^{nc} - E(L_M)_{IR}^c = \frac{1}{2} [1 + \alpha_M (\phi_{II}^R)^2] \{V\var(\pi)_{IR}^{nc} - V\var(\pi)_{IR}^c\} \tag{C.4}
\]

The reaction and the reference parameters for both fiscal regimes are taken from Table 1.

C.2. Proof of Proposition 3

For the simultaneous-move strategic regime, we get:

\[
E \left(L_{F_1}\right)_{SM}^{nc} - E \left(L_{F_1}\right)_{SM}^c = \frac{1}{2} \frac{\alpha_F (\phi_{II}^SM)^2 (\omega_i^2 \sigma_i^2 + \delta_i^2 \sigma_i^2)}{(\omega_i^2 + \delta_i^2)^2} [1 + \alpha_F (Z_g - Z_g^*)]^2 \left[1 + \alpha_F (Z_g + Z_g^*)^2 \right] \left(\Omega_{SM}^{nc} \Omega_{SM}^c \right)^2 \left(\Omega_{SM}^{nc} \Omega_{SM}^c \right)^2.
\]
where we can get a clear-cut result for \( \delta_g \omega_i + \delta_r \omega_g = 0 \), where \( E \left( L_{F_I} \right)_{SM}^{nc} - E \left( L_{F_I} \right)_{SM}^c = \frac{1}{2} \frac{a_F Z_g (\phi_{SM}^c)^2 (\omega_i^2 \sigma_i^2 + \delta_r^2 \sigma_r^2)}{\delta_r^2 [1 + a_M (\phi_{SM}^c)^2]^2} \left( (Z_g - Z_g^*)^2 + a_F (Z_g - Z_g^*) (2Z_g + Z_g^*) + (2Z_g + Z_g^*) \right) \left( a_F Z_g (Z_g - Z_g^*)^2 \right) < 0 \), as \( Z_g - Z_g^* > 0 \). For the fiscal leadership regime, we get:

\[
E \left( L_{F_I} \right)_{FL}^{nc} - E \left( L_{F_I} \right)_{FL}^c = \frac{1}{2} \frac{(\phi_{SM}^c)^2 (\omega_i^2 \sigma_i^2 + \delta_r^2 \sigma_r^2)}{\left( \frac{\Omega_{FL}^c}{\Omega_{FL}^c} \right)^2} \left( a_F (\Omega_{FL}^c - \Omega_{FL}^c) (\Omega_{FL}^c + \Omega_{FL}^c) + (\phi_{gnc}^c \Omega_{FL}^c - \phi_{gnc}^c \Omega_{FL}^c) + \phi_{gnc}^c \Omega_{FL}^c + \phi_{gnc}^c \Omega_{FL}^c) \right)
\]

where we already know \( \Omega_{FL}^c - \Omega_{FL}^c \) and \( \Omega_{FL}^c + \Omega_{FL}^c \) from the Proof of Proposition 2. Moreover, \( \phi_{gnc}^c \Omega_{FL}^c - \phi_{gnc}^c \Omega_{FL}^c = -a_F \left( Z_g^* - \frac{1}{2} Z_i V_i \right) \delta_r [1 + a_M (\phi_{SM}^c)^2] \) and \( \phi_{gnc}^c \Omega_{FL}^c + \phi_{gnc}^c \Omega_{FL}^c = a_F \left\{ 2 \left( Z_g - \frac{1}{2} Z_i V_i \right) \Omega_{FL}^c + \left( Z_g^* - \frac{1}{2} Z_i V_i \right) \delta_r [1 + a_M (\phi_{SM}^c)^2] \right\} \). We can see that we can only get an unambiguous result for both \( (\delta_g \omega_i + \delta_r \omega_g) (Z_g^* - \frac{1}{2} Z_i V_i) > 0 \) and \( (Z_g - \frac{1}{2} Z_i V_i) (Z_g^* - \frac{1}{2} Z_i V_i) > 0 \), where \( E \left( L_{F_I} \right)_{FL}^{nc} < E \left( L_{F_I} \right)_{FL}^c \).

Thus, for the special case of \( \delta_g \omega_i + \delta_r \omega_g = 0 \), we need \( (Z_g - \frac{1}{2} Z_i V_i) (Z_g^* - \frac{1}{2} Z_i V_i) > 0 \).

C.3. Proof of Proposition 5

For both the simultaneous-move and the fiscal leadership regime, we compare society’s loss under the two alternative fiscal regimes, as:

\[
E \left( L_{SM} \right)_{IR}^{nc} - E \left( L_{SM} \right)_{IR}^c = \frac{1}{2} \frac{\omega_i^2 \text{Var}(u) + \delta_r^2 \text{Var}(e)}{\left( \frac{\Omega_{IR}^c}{\Omega_{IR}^c} \right)^2} \left\{ [1 + a_M (\phi_{SM}^c)^2] (\Omega_{IR}^c - \Omega_{IR}^c) (\Omega_{IR}^c + \Omega_{IR}^c) + b_S (\phi_{SM}^c)^2 (\phi_{gnc}^c \Omega_{IR}^c - \phi_{gnc}^c \Omega_{IR}^c) (\phi_{gnc}^c \Omega_{IR}^c + \phi_{gnc}^c \Omega_{IR}^c) \right\}
\]

For the simultaneous-move strategic regime, we get:

\[
E \left( L_{SM} \right)_{SM}^{nc} - E \left( L_{SM} \right)_{SM}^c = -\frac{1}{2} \frac{a_F Z_g (\phi_{SM}^c)^2 (\omega_i^2 \text{Var}(u) + \delta_r^2 \text{Var}(e))}{\left( \frac{\Omega_{SM}^c}{\Omega_{SM}^c} \right)^2} \left( a_F (Z_g + Z_g^*) \phi_{SM}^c * \left\{ b_S \delta_r^2 (1 + a_M (\phi_{SM}^c)^2) - \right\} \right) \left\{ (\delta_g \omega_i + \delta_r \omega_g) \right\} \left( 1 + a_M (\phi_{SM}^c)^2 \right) + 2 \delta_r [1 + a_M (\phi_{SM}^c)^2] \left( \delta_g \omega_i + \delta_r \omega_g \right) * \left[ 1 + a_M (\phi_{SM}^c)^2 \right] \left( Z_g + Z_g^* \right) (\phi_{SM}^c)^2 \] \left( \omega_i^2 \text{Var}(u) + \delta_r^2 \text{Var}(e) \right) < 0 \)

For the special case of \( \delta_g \omega_i + \delta_r \omega_g = 0 \), we get:

\[
E \left( L_{SM} \right)_{SM}^{nc} - E \left( L_{SM} \right)_{SM}^c = -\frac{1}{2} \frac{b_S a_F Z_g (Z_g + Z_g^*) \phi_{SM}^c (\omega_i^2 \text{Var}(u) + \delta_r^2 \text{Var}(e))}{\left( \frac{\Omega_{SM}^c}{\Omega_{SM}^c} \right)^2} \left( 1 + a_M (\phi_{SM}^c)^2 \right) < 0 \]

\sim 43 \sim
D. Comparison of the Alternative Strategic Regimes (Section 6)

D.1. The Simultaneous-Move versus the Fiscal Leadership Regime

We compare the union-wide inflation rate and the output gap, following equilibrium solutions (equations 15 and 16). We get:

\[ \text{Var}(\pi)_{SM} - \text{Var}(\pi)_{FL} = \frac{\omega^2 \text{Var}(u) + \delta^2 \text{Var}(\varepsilon)}{\Omega_{SM} \Omega_{FL}} \ast (\Omega_{FL} - \Omega_{SM})(\Omega_{FL} + \Omega_{SM}) \]  

(D.1)

\[ \text{Var}(y)_{SM} - \text{Var}(y)_{FL} = (\phi^S_{SM})^2 [\text{Var}(\pi)_{SM} - \text{Var}(\pi)_{FL}], \]  

(D.2)

where \( \Omega_{FL} - \Omega_{SM} = (\delta_g \omega_i + \delta_r \omega_g)\phi^S_{SM} (\phi_g^S - \phi_g^F) \) and \( \Omega_{FL} + \Omega_{SM} = 2\delta_r [1 + \alpha_M (\phi^S_{\pi})^2] - (\delta_g \omega_i + \delta_r \omega_g)\phi^S_{SM} (\phi_g^S + \phi_g^F). \) For the two fiscal regimes, we get:

\[ \begin{align*}
\text{Var}(\pi)_{SM}^c - \text{Var}(\pi)_{FL}^c & = \frac{\omega^2 \text{Var}(u) + \delta^2 \text{Var}(\varepsilon)}{\Omega_{SM}^c \Omega_{FL}^c} a_F Z_i \Phi(\delta_g \omega_i + \delta_r \omega_g) [\delta_r [1 + \alpha_M (\phi^S_{\pi})^2] - (\delta_g \omega_i + \delta_r \omega_g)\phi^S_{SM} a_F \left(Z_g - \frac{1}{4} Z_i V_i \right)] \\
\text{Var}(y)_{SM}^c - \text{Var}(y)_{FL}^c & = (\phi^S_{SM})^2 \text{Var}(\pi)_{SM} - (\phi^S_{SM})^2 \text{Var}(\pi)_{FL}
\end{align*} \]  

(D.3)

Both equations (D.3) and (D.4) reveal that in the special case of \( \delta_g \omega_i + \delta_r \omega_g = 0, \) then \( \text{Var}(\pi)_{SM} = \text{Var}(\pi)_{FL} \) and \( \text{Var}(y)_{SM} = \text{Var}(y)_{FL}. \) Following equation (D.3), under \( \delta_g \omega_i + \delta_r \omega_g > 0 \) and \( Z_g - \frac{1}{4} Z_i V_i < 0, \) then \( \text{Var}(\pi)_{SM}^c > \text{Var}(\pi)_{FL}^c, \) and vice versa. Following equation (D.4) for the cooperative fiscal regime, under \( \delta_g \omega_i + \delta_r \omega_g > 0 \) and \( 2(Z_g + Z_i^*) - Z_i V_i < 0, \) then \( \text{Var}(\pi)_{SM} > \text{Var}(\pi)_{FL}^c. \) However, under \( \delta_g \omega_i + \delta_r \omega_g < 0 \) then \( 2(Z_g + Z_i^*) - Z_i V_i = \delta_g \delta_r [1 + \alpha_M (\phi^S_{\pi})^2] > 0. \) This means that \( \text{Var}(\pi)_{SM} < \text{Var}(\pi)_{FL}^c, \) hence \( \text{Var}(y)_{SM} < \text{Var}(y)_{FL}^c. \)

Turning now to the union-wide fiscal stance, we get:

\[ \text{Var}(g)_{SM} - \text{Var}(g)_{FL} = \frac{(\phi^S_{\pi})^2 [\omega^2 \text{Var}(u) + \delta^2 \text{Var}(\varepsilon)] (\phi_g^S \Omega_{FL} - \phi_g^F \Omega_{FL})(\phi_g^S \Omega_{FL} + \phi_g^F \Omega_{FL})}{(\Omega_{SM} \Omega_{FL})^2} \]  

(D.5)

For the decentralized fiscal regime, \( \phi_g^S \Omega_{FL}^c - \phi_g^F \Omega_{FL}^c = \frac{1}{2} a_F \delta_r Z_i V_i [1 + \alpha_M (\phi^S_{\pi})^2] > 0 \) and \( \phi_g^S \Omega_{FL}^c + \phi_g^F \Omega_{FL}^c = a_F \left(2Z_g - \frac{1}{2} Z_i V_i \right) \delta_r [1 + \alpha_M (\phi^S_{\pi})^2] - 2(\delta_g \omega_i + \delta_r \omega_g)\phi^S_{SM} a_F Z_g (Z_g - \frac{1}{4} Z_i V_i). \)
Thus, under $\delta_g \omega_i + \delta_r \omega_g = 0$, then $\text{sign}\{\text{Var}(g)_{SM}^{nc} - \text{Var}(g)_{FL}^{nc}\} = \text{sign}\left\{2Z_g - \frac{1}{2}Z_iV_i\right\}$, which is unambiguously positive for $\omega_g = \omega_i = 0$. For $\text{Var}(g)_{SM}^{nc} > \text{Var}(g)_{FL}^{nc}$ we need either $Z_g - \frac{1}{2}Z_iV_i > 0$ or $Z_g - \frac{1}{2}Z_iV_i < 0$ and $2Z_g - \frac{1}{2}Z_iV_i < 0$ for $\delta_g \omega_i + \delta_r \omega_g < 0$, and both $Z_g - \frac{1}{2}Z_iV_i < 0$ and $2Z_g - \frac{1}{2}Z_iV_i > 0$ for $\delta_g \omega_i + \delta_r \omega_g > 0$. For the centralized fiscal regime, we have $\phi_{g_c}^{SM} \Omega_{FL}^c - \phi_{g_c}^{FL} \Omega_{SM}^c = a_F \delta_g Z_iV_i [1 + a_M(\phi_{\pi}^{SM})^2] > 0$ and $\phi_{g_c}^{SM} \Omega_{FL}^c + \phi_{g_c}^{FL} \Omega_{SM}^c = a_F \left[2(Z_g + Z_i^* - Z_iV_i)\delta_r [1 + a_M(\phi_{\pi}^{SM})^2] + 2\phi_{g_c}^{SM} (\delta_g \omega_i + \delta_r \omega_g) (\phi_{\pi}^{SM})^2\right]$. We can get a clear-cut result for $\delta_g \omega_i + \delta_r \omega_g < 0$, where $2Z_g + 2Z_i^* - Z_iV_i > 0$, hence $\text{Var}(g)_{SM}^c > \text{Var}(g)_{FL}^c$. In the special case of $\delta_g \omega_i + \delta_r \omega_g = 0$, then $\phi_{g_c}^{SM} \Omega_{FL}^c + \phi_{g_c}^{FL} \Omega_{SM}^c = a_F \delta_g [1 + a_M(\phi_{\pi}^{SM})^2] \frac{1}{1 - \delta_g} > 0$, which again leads to $\text{Var}(g)_{SM}^c > \text{Var}(g)_{FL}^c$.

D.2. Proof of Proposition 6

The social planner’s expected loss can be computed from equation (9) as:

$$E(L_S) = \frac{1}{2} * \frac{1 + (a_g + b_g \phi_{\pi}^2)}{\Omega^2} * \left[\omega_i^2 \text{Var}(u) + \delta_g^2 \text{Var}(\epsilon)\right]$$

We compare the expected losses for the two strategic regimes, as:

$$E(L_S)_{SM} - E(L_S)_{FL} = \frac{1}{2} * \frac{\omega_i^2 \text{Var}(u) + \delta_g^2 \text{Var}(\epsilon)}{(\Omega_{SM} \Omega_{FL})^2} \left[1 + a_g (\phi_{\pi}^{SM})^2 \right] (\Omega_{FL} - \Omega_{SM})(\Omega_{FL} + \Omega_{SM}) +$$

$$b_g (\phi_{\pi}^{SM})^2 (\phi_{g_c}^{SM} \Omega_{FL} - \phi_{g_c}^{FL} \Omega_{SM}) (\phi_{g_c}^{SM} \Omega_{FL} - \phi_{g_c}^{FL} \Omega_{SM})$$

The above equation presents the general formula for both fiscal regimes. The specific formula for the decentralized fiscal regime is given by:

$$E(L_S)_{SM}^{nc} - E(L_S)_{FL}^{nc} = \frac{1}{2} * \frac{\omega_i^2 \text{Var}(u) + \delta_g^2 \text{Var}(\epsilon)}{(\Omega_{SM} \Omega_{FL})^2} * a_F \phi_{\pi}^{SM} Z_iV_i \left\{[\delta_g \omega_i + \delta_r \omega_g][1 + a_g (\phi_{\pi}^{SM})^2] \right\}$$

$$\left[\delta_r [1 + a_M(\phi_{\pi}^{SM})^2] - (\delta_g \omega_i + \delta_r \omega_g) a_F \phi_{\pi}^{SM} \left(Z_g - \frac{1}{4}Z_iV_i\right) + b_M a_F \phi_{\pi}^{SM} \delta_r [1 + a_M(\phi_{\pi}^{SM})^2] \left[Z_g - \frac{1}{4}Z_iV_i\right] \delta_r [1 + a_M(\phi_{\pi}^{SM})^2] - (\delta_g \omega_i + \delta_r \omega_g) a_F \phi_{\pi}^{SM} Z_g \left(Z_g - \frac{1}{4}Z_iV_i\right) \right\}.$$ 

Under $\delta_g \omega_i + \delta_r \omega_g = 0$, then $\text{sign}\{E(L_S)_{SM}^{nc} - E(L_s)_{FL}^{nc}\} = \text{sign}\left\{Z_g - \frac{1}{4}Z_iV_i\right\}$, which is unambiguously positive for $\omega_g = \omega_i = 0$. Thus, $E(L_S)_{SM}^{nc} > E(L_s)_{FL}^{nc}$. For centralized fiscal policies, we get:
\[E(L_S)^c_{SM} - E(L_S)^c_{FL} = \frac{1}{2} \frac{\omega^2 \text{Var}(\omega) + \delta^2 \text{Var}(\epsilon)}{(\delta g^2 \omega + \delta r \omega)^2} \cdot a_F \phi_{SM}^2 Z \cdot V_i \left\{ (\delta g \omega + \delta r \omega)[1 + a_M(\phi_{SM}^2)] \right. \]

\[a_S(\phi_{SM}^2)[2\delta_r[1 + a_M(\phi_{SM}^2)] - (\delta g \omega + \delta_r \omega) a_F \phi_{SM}^2 (2Z_g + 2Z_g^* - Z_i V_i)] + b_S a_F \phi_{SM}^2 \delta_r[1 + a_M(\phi_{SM}^2)] \cdot \left. \right\} \]

which again can be of either sign for any \(\delta_g \omega + \delta_r \omega \geq 0\). Under \(\delta_g \omega + \delta_r \omega = 0\), then

\[\text{sign}\{E(L_S)^c_{SM} - E(L_S)^c_{FL}\} = \text{sign}\{2Z_g + 2Z_g^* - Z_i V_i = \frac{\delta g}{1-\delta_g}\},\]

which is unambiguously positive. Thus, \(E(L_S)^c_{SM} > E(L_S)^c_{FL}\).

\[\text{D.3. Proof of Proposition 7}\]

The general case can be computed as:

\[E(L_S)^c_{FL} - E(L_S)^c_{OC} = \frac{1}{2} \frac{\omega^2 \text{Var}(\omega) + \delta^2 \text{Var}(\epsilon)}{(\delta g^2 \omega + \delta r \omega)^2} \cdot a_F \phi_{SM}^2 \phi_{OC}^2 \cdot \left\{ (\omega_{OC} - \Omega_{OC}) + \Omega_{FL} + \omega_{OC} - a_M \Omega_{FL} \right\} \]

Let \(\delta_g \omega + \delta_r \omega = 0\). Then, the previous equation becomes:

\[E(L_S)^c_{FL} - E(L_S)^c_{OC} = \frac{1}{2} \frac{\omega^2 \text{Var}(\omega) + \delta^2 \text{Var}(\epsilon)}{(\delta g^2 \omega + \delta r \omega)^2} \cdot a_F \phi_{SM}^2 \phi_{OC}^2 \cdot \left\{ (a_F - a_M) \omega_{OC} + a_M \Omega_{FL} \right\} \]

(i) For \(a_S = a_F = a_M = a\) then the relation in brackets becomes negative, and equal to \(-a^2(3 + 8a(\phi_{OC}^2))\). This leads to \(E(L_S)^c_{FL} < E(L_S)^c_{OC}\).

(ii) For \(a_S = a_F + a_M\), then the relation in brackets becomes \((a_F + a_M)(a_F - 2a_M)[1 + (\phi_{OC}^2)]\). Thus, \(a_F - 2a_M \geq 0 \Rightarrow E(L_S)^c_{FL} \geq E(L_S)^c_{OC}\).

(iii) For \(a_S = a_M\) and \(a_F < a_M\), then the relation in brackets becomes negative, equal to \(-\{a_M(a_F + 2a_M) - (a_F + a_M)(\phi_{OC}^2) \cdot \left\{ a_M(a_M - a_F) + (a_F + a_M)^2 \right\}\}. This again leads to \(E(L_S)^c_{FL} < E(L_S)^c_{OC}\).
References


~ 50 ~


